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RACE DIFFERENCES IN THE ORGANIZATION OF NUMERICAL AND VERBAL ABILITIES

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FOR STUDIES IN EDUCATION

BY
JACK W. DUNLAP, Ph.D.

ARCHIVES OF PSYCHOLOGY
R. S. WOODWORTH, Editor

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ONTARIO COLLEGE OF EDUCATION
DEPT. OF EDUCATIONAL RESEARCH
UNIVERSITY OF TORONTO

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Any errors in setting up the problem, in the statistical development, or in interpretation can be attributed only to the writer.

J. W. D.

TABLE OF CONTENTS

Chapter *Page*

I. The Problem . . . 7

II. The Stocks and The Tests . . . 9

III. Differences in Central Tendency and Variability . . . 14

IV. The Organization of Abilities in the Six Stocks . . . 32

V. The Idiosyncrasies of Stocks in the Comparative Magnitudes of Numerical and Verbal Abilities . . . 46

APPENDIX

I. Basic Tables of Means, Standard Deviations, and Intercorrelations . . . 51

II. Tables of Partial Correlations Used in Computing the Tetrads . . . 55

III. The Standard Error of the Mean of Tetrads of the Type t_{12ij} . . . 59

IV. The Variance of a Tetrad Ratio and the Covariance between Tetrad Ratios from the Same Four Variables . . . 66

V. The Analysis of a Particular Pattern for Five Variables 68

VI. Sex Differences and Idiosyncrasies . . . 70

VII. Citations . . . 72

Race Differences in the Organization of Numerical and Verbal Abilities

CHAPTER I

THE PROBLEM

This study will attempt to answer the following questions:

1. What differences in test scores in numerical and verbal material are found between the stocks here studied? In numerical and verbal material what sex differences are found within stocks?

2. Does the group factor found for numerical ability by Kelley and others appear in the several stocks?

3. How does the relative proportion of numerical ability to verbal ability vary from stock to stock?

This study is limited to the problem of differences in numerical ability and verbal ability by the racial groups, between the ages of ten years, no months, and thirteen years, eleven months, of Japanese, Chinese, Portuguese, Hawaiian, Part-Hawaiian, Korean, and Filipino children in the public schools of Honolulu, Hawaii, in the spring of 1927.

Certain of the tests will be referred to as number tests, and certain other tests as verbal tests. Verbal ability will be thought of as performance in those tests which require predominately a knowledge of words and the ability to manipulate them in various concepts. Numerical ability will be thought of as performance in those tests which involve for the most part the ability to operate with numbers and number concepts.

The "races" or "racial groups" or "natio-racial groups" which for convenience will be called stocks, are defined by records of the parental and grandparental generations. The linguistic difficulties are eliminated or greatly reduced. The climate is substantially the same for all the stocks studied. The sampling of the Hawaiian group is impartial. The other stocks represent samplings from their respective source stocks by approximately the same forces (namely immigration to work on the sugar plantations). The socio-economic level is relatively constant from stock to stock. These factors must be recognized and accounted for as far as possible in reducing heterogeneity.

CHAPTER II

The Stocks and the Tests

The public school population of Honolulu, grades three to eight inclusive, were tested on the Stanford Achievement Test.* The white stocks were discarded as considerable selection entered, due to the tendency of the white parents to send their children to private schools.

The Japanese, Chinese, and Portuguese children studied represent, for the most part, third and fourth generations of these stocks in the islands. The Part-Hawaiians are mostly second and third generation in the islands on the non-Hawaiian side.** The Korean and Filipino children represent primarily the first generation born in the islands of these two stocks.*** History indicates that the Hawaiians had been in Hawaii almost ten centuries before the islands were discovered by Cook in 1778. During this period the Hawaiians developed as a homogeneous subrace.

The historical background of the present generation of stocks was as follows: the parent or grandparent generations of five of these stocks were imported or came of their own volition as field workers for the large sugar plantations. As the members of such a group achieved economic independence they usually left the plantation and moved into the city as independent tradesmen or as skilled or semi-skilled workers in mercantile or industrial establishments. The exodus of labor from plantation to town required more laborers to do the unskilled work of the plantations. For economic, political, and social reasons the source of labor supply shifted from time to time.

The Japanese, Chinese, and Portuguese have now for the most part left the unskilled labor of the plantations for a higher economic standing in town. The Koreans, a new group in terms of time in the islands, have also achieved economic independence and are now found concentrated in the larger population centers. The Filipinos were the last group to come and still are primarily a plantation group. The Part-Hawaiians and the Hawaiians have never been successfully used in the plantation fields and are found in positions of more or less authority. Many have their own land and get a livelihood from that source.

---

* Except about 500 children, the addition of which would not have altered the nature of the sampling in any important way.

** The Part-Hawaiians are a more heterogeneous group than the other six stocks but are kept for comparative purposes.

*** The Filipinos are too few to allow inferences statistically sound to be drawn. They represent, however, the entire population and are kept as they are better than nothing.

Historically five of the stocks represent labor importations. It might be assumed that those willing to venture across the seas represent a hardier and stronger group than those who remained in their ancestral land. It seems fair to assume that a selective factor of some nature has narrowed the range of the groups from what one would discover in a random sampling of the source stocks. The Koreans probably represent two types of sampling. One group of them were political refugees, while the other group came as laborers.

The Hawaiians have intermingled freely with the new races. No apparent social barrier seems to have been set up in their case at any time. The Chinese, the oldest group in terms of time in the islands, have intermarried with later migrants to some extent, and even more so with the Hawaiians. The Japanese have kept their group quite pure, but during the last decade there is evidence of breakdown of group barriers. The Portuguese have intermarried with the Hawaiians and to a lesser degree with the Chinese. The Koreans have been to date a comparatively closed group, but like the Japanese, are now tending to interalliance. The Filipinos have been considered socially inferior, but this attitude will probably change in time. Except for the last mentioned fact, no important social barriers exist between the stocks.

Economically these groups are on approximately the same level. The Chinese have many small shopkeepers, the Japanese practically control the fishing, while the Portuguese are in clerical work and in positions of minor authority. The Hawaiians and Part-Hawaiians in Honolulu hold nearly all the elective official and semi-official public offices. There are outstanding men in each group, just as there are very poor ones.

In the legislature Hawaiians, Part-Hawaiians, Chinese, Japanese and Whites are found. The Korean group is too small and the Filipino too new yet, to hope to enter this field.

The teachers within a school may be Chinese, Japanese, Hawaiian, Part-Hawaiian, Portuguese, Korean, White, or some mixture.* Thus a child in school is exposed to the influence of teachers from the various stocks. In a given classroom one practically always will find representatives from three of the stocks and usually many more, as the stocks studied are by no means all of the natio-racial groups to be found in the public schools of Honolulu.

The seating arrangement in a school room has no regard for the racial grouping. In forming a school's athletic and other teams, stock in itself is given no consideration. In athletics, however, certain groups predominate in certain sports, for instance the Hawaiian in football, due to superior height and weight.

English is the common language of the islands. Many if not most of the children are, however, more or less bi-lingual. Some are able to understand only words and phrases of their ancestral language while others have a real command of its speech and a more or less limited facility in its written language. The auxiliary language schools supported by the various stocks are vanishing as they are no longer of value. These schools which gave or are still giving instructions in the mother tongue have been patronized more tenaciously by the Japanese than by other stocks. The Chinese and Portuguese also have had separate language schools but nearly all have lapsed.

The Hawaiian islands are located in so small an area that climatic conditions are practically constant, so that a migration from one of the other islands to Honolulu would not appreciably modify the effect of climate or geography.

Homogeneity is essential when a study of differences between stocks is made. This investigation, however, is interested in homogeneity from a different point of view. In order to test theories of the organization of ability into elements or factors, homogeneity of the data is necessary. Also, in order to discover the differences between groups in organization of abilities homogeneity of each group is important. To secure complete homogeneity of stocks with regard to the complex interactions of mores, economics, and language is difficult and probably can never be accomplished. Heterogeneity of stocks, nevertheless, must be reduced to a minimum. The principles of selection for homogeneity were applied to ancestry, age, number of years in school, and knowledge of English.

---

* One case, called to my attention, boasted of nine different blood strains, and another of seven.

A child shall be classified in any one stock if the two parents and four grandparents are from that stock, except that a child having Hawaiian blood mixed with any other or others in the two degrees of descent shall be classified as Part-Hawaiian. The parentage and the second degree of descent were given by the child's birth certificate which was recorded when the child entered school. Using the above definition the children were classified into six stocks (Japanese, Chinese, Portuguese, Hawaiian, Korean, and Filipino), and one mixture (Part-Hawaiian).

Only those children who were between the ages ten years, no months and thirteen years, eleven months were selected.*

It is recognized that bilingualism introduces heterogeneity, and to reduce this effect only children who have been in school at least three years are used. English is the language of the streets and playgrounds as well as of the school, so the children are presumably English thinking as well as English speaking.

This study, hence, deals with children in a four-year age range homogeneous as to stock to the second degree of descent. Climate, mores, and economic status represent for the stocks studied, conditions of relative constancy.

The Stanford Achievement Test, Form A, had been administered to all these children in the spring of 1927.** The subtests on which the selected groups were compared were Arithmetic Computation, Arithmetic Reasoning, Paragraph Meaning, Sentence Meaning, and Word Meaning.

---

* By using this group practically all of the children at those ages in Honolulu were secured. The upper level is well within the compulsory school limit, while only a slight selective error has entered in that some of the brighter pupils had already entered high school.

Prepubertal and pubertal groups have been mixed but probably not to such an extent as would be the case on the mainland of the United States, as children in Hawaii in general reach maturity somewhat earlier. Apparently, no serious error has been introduced by so doing, for there are no sex differences which are generally in favor of the females. In fact the only sex differences are in favor of the males.

** These data were made available through the courtesy of Mr. G. A. Webling, Supervising Principal of the elementary schools of Honolulu. The principals of the various schools kindly made the school records available so that age and race could be verified.

The reliability of these subtests for the ages 10, 11, 12, and 13, as given by Kelley, Ruch, and Terman (1924) are Arithmetic Computation, .92, .89, .85, .85; Arithmetic Reasoning, .88, .87, .90, .91; Paragraph Meaning, .94, .92, .90, .91; Sentence Meaning, .87, .89, .88, .89; and Word Meaning, .96, .92, .94, .96.*

The high reliability of these tests is important as the data will be treated later by refined statistical techniques.

---

* Miss Ida Caro (1929) and Miss Gladys Traut (1929), in unpublished theses for the master's degree at the University of Hawaii, found substantially the same values, in all cases when calculated on children born in Hawaii.

CHAPTER III

DIFFERENCES IN CENTRAL TENDENCY AND VARIABILITY

This study deals with the six stocks and one mixture which are presented in the following order: Japanese, Chinese, Portuguese, Hawaiian, Part-Hawaiian, Korean, and Filipino.* The term stock differences is used in comparing the mean scores of two stocks on a particular test. The term sex differences is used in comparing the mean score of the males with the mean score of the females of a particular stock on a specified test.

On each subtest the mean for each sex of each stock was computed. The difference between means and the statistical significance of these differences were obtained. For example, the difference between the mean of scores in Arithmetic Computation and the probable error of this difference, for Japanese males and Chinese males was determined.**

$$\sigma^2_{(M_1 - M_2)} = \sigma^2_{M_1} + \sigma^2_{M_2}$$

Arithmetic Computation

Tables III, IV, and V, together with line graphs 1 and 2, present the data and summary for the various stock and sex groups. These three tables and two graphs together make a basic unit of information.

Table III gives for Arithmetic Computation the fundamental numerical data and the derived values with their probable errors. Graphs 1 and 2 make it possible to see at a glance the ranking and scores of a particular stock sex group for Arithmetic Computation. Tables IV and V show the stocks that a given stock significantly exceeds in mean scores. The other sets of tables in this section give similar information.

---

* The stocks are arranged in order of size except that the Part-Hawaiian group is placed after the Hawaiian group as it is an indeterminate *pot pourri* of some measure of Hawaiian blood and various complements of other stocks.

** The variance of the difference between means of any two groups, as there is no correlation, is given by the formula

The first row of Table IV is read as follows: The mean score made by Chinese males in Arithmetic Computation is significantly* greater than the mean score of the Japanese, Filipino, Part-Hawaiian, Portuguese, and Hawaiian males. The remaining rows in the table are similarly interpreted.

---

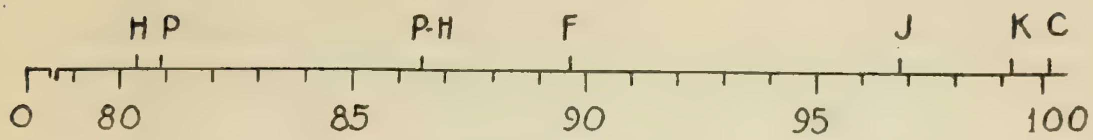
*When a difference is three or more times its probable error it is termed significant.

TABLE II

DIFFERENCES BETWEEN MEANS WITH THE PROBABLE ERRORS OF THOSE DIFFERENCES FOR ARITHMETIC COMPUTATION*

Males	*Females*			*Jap*	*Chi.*	*Port.*	*Haw.*	*P. Haw.*	*Kor.*	*Fil.*
			M	*96.41*	*98.46*	*78.80*	*82.51*	*88.46*	*98.77*	*99.03*
			σ	*23.89*	*24.06*	*24.78*	*24.06*	*21.87*	*26.36*	*26.00*
	M	*σ*	*N*	*843*	*453*	*280*	*63*	*244*	*52*	*31*
Japanese	96.82	25.05	876	**.41**	−2.05	17.61	13.90	7.95	− 2.36	− 2.62
				± .80	± .94	± 1.14	± 2.12	± 1.09	± 2.53	± 3.20
Chinese	99.97	23.95	442	−3.15	**1.51**	19.66	15.95	10.00	− .31	− .57
				± .96	**±1.09**	± 1.25	± 2.19	± 1.21	± 2.58	± 3.24
Portuguese	80.89	25.95	139	15.93	19.08	**2.09**	− 3.71	− 9.66	−19.97	−20.23
				±1.59	±1.67	**± 1.79**	± 2.27	± 1.38	± 2.66	± 3.31
Hawaiian	80.42	25.78	81	16.40	19.55	.47	**− 2.09**	− 5.95	−16.26	−16.52
				±2.02	±2.08	± 2.43	**± 2.81**	± 2.25	± 3.20	± 3.76
Pt. Hawaiian	86.46	22.56	219	10.36	13.51	− 5.57	− 6.04	**− 2.00**	−10.31	−10.57
				±1.17	±1.28	± 1.81	± 2.19	**± 1.40**	± 2.64	± 3.28
Korean	99.16	25.13	67	−2.34	.81	−18.27	−18.74	−12.70	**.38**	− .26
				±2.23	±2.29	± 2.62	± 2.89	± 2.39	**± 3.27**	± 4.00
Filipino	89.68	21.64	25	7.14	10.29	− 8.79	− 9.26	− 3.22	9.48	**− 9.35**
				±2.97	±3.02	± 3.28	± 3.50	± 3.06	± 3.63	**± 4.30**

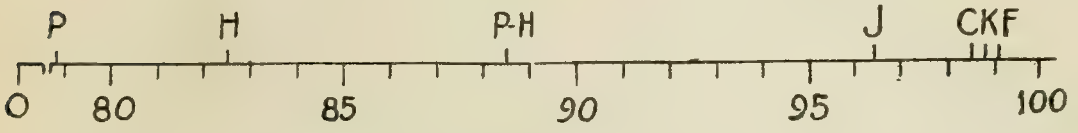
* Bold face figures in the diagonal are sex differences. Figures above and to the right of the diagonal give the stock differences for females and those below and to the left of the diagonal give the stock differences for the males. Differences for the males are obtained by subtracting from the mean of the race of a given column, successively, the values of the means of the races in the rows. Differences for the females are obtained by subtracting from the mean of the race of a given row successively the means of the races in the columns. The means, standard deviations, and populations are also given.



GRAPH 1
MEAN SCORES IN ARITHMETIC COMPUTATION FOR MALES

TABLE IV
GROUPS (MALES) WHOSE MEANS IN ARITHMETIC COMPUTATION ARE 3 P. E. OR MORE BELOW THOSE OF THE RACE IN THE LEFT HAND COLUMN

	C	*K*	*J*	*F*	*P-H*	*P*	*H*
Chinese			J	F	P-H	P	H
Korean					P-H	P	H
Japanese					P-H	P	H
Filipino							
Pt. Hawaiian						P	
Portuguese							
Hawaiian							



GRAPH 2
MEAN SCORES IN ARITHMETIC COMPUTATION FOR FEMALES

TABLE V
GROUPS (FEMALES) WHOSE MEANS IN ARITHMETIC COMPUTATION ARE 3 P. E. OR MORE BELOW THOSE OF THE RACE IN THE LEFT HAND COLUMN

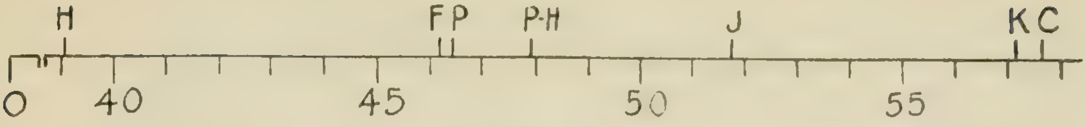
	F	*K*	*C*	*J*	*P-H*	*H*	*P*
Filipino					P-H	H	P
Korean					P-H	H	P
Chinese					P-H	H	P
Japanese					P-H	H	P
Pt. Hawaiian							P
Hawaiian							
Portuguese							

TABLE VI

DIFFERENCES BETWEEN MEANS WITH THE PROBABLE ERRORS OF THOSE DIFFERENCES FOR ARITHMETIC REASONING *

Males	*Females*			*Jap.*	*Chi.*	*Port.*	*Haw.*	*P. Haw.*	*Kor.*	*Fil.*
			M	*45.07*	*50.97*	*40.93*	*33.49*	*42.98*	*51.08*	*44.84*
			σ	*17.83*	*19.75*	*17.39*	*13.81*	*16.04*	*24.36*	*21.84*
	M	*σ*	*N*	*843*	*453*	*280*	*63*	*244*	*52*	*31*
Japanese	51.68	19.69	876	**6.61**	−5.85	4.14	11.58	2.09	− 6.01	.23
				± .61	± .75	± .82	± 1.24	± .81	± 2.31	± 2.68
Chinese	57.57	20.93	442	−5.89	**6.65**	9.99	17.43	7.94	− .16	6.08
				± .81	**± .92**	± .94	± 1.33	± .93	± 2.36	± 2.72
Portuguese	46.37	17.49	139	5.31	11.20	**5.44**	7.44	− 2.05	−10.15	− 3.91
				±1.09	±1.21	**± 1.22**	± 1.37	± .98	± 2.38	± 2.74
Hawaiian	39.04	19.25	81	12.64	18.53	7.33	**5.55**	− 9.49	−17.59	−11.35
				±1.51	±1.59	± 1.75	**± 1.86**	± 1.36	± 2.56	± 2.89
Pt. Hawaiian	47.88	17.89	219	3.80	9.69	− 1.51	− 8.84	**4.90**	− 8.10	− 1.86
				± .93	±1.06	± 1.29	± 1.66	**± 1.07**	± 2.38	± 2.73
Korean	57.10	18.35	67	−5.42	.47	−10.73	−18.06	− 9.22	**6.02**	6.24
				±1.63	±1.71	± 1.81	± 2.13	± 1.77	**± 2.77**	± 3.49
Filipino	46.16	17.54	25	5.52	11.41	.21	− 7.12	1.72	10.94	**1.32**
				±2.41	±2.92	± 2.57	± 2.77	± 2.50	± 2.84	**± 3.55**

* Bold face figures in the diagonal are sex differences. Figures above and to the right of the diagonal give the stock differences for females and those below and to the left of the diagonal give the stock differences for the males. Differences for the males are obtained by subtracting from the mean of the race of a given column, successively, the values of the means of the races in the rows. Differences for the females are obtained by subtracting from the mean of the race of a given row successively the means of the races in the columns. The means, standard deviations, and populations are also given.



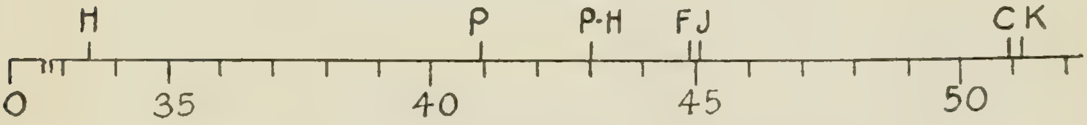
GRAPH 3

MEAN SCORES IN ARITHMETIC REASONING FOR MALES

TABLE VII

GROUPS (MALES) WHOSE MEANS IN ARITHMETIC REASONING ARE 3 P. E. OR MORE BELOW THOSE OF THE RACE IN THE LEFT HAND COLUMN

	C	*K*	*J*	*P-H*	*P*	*F*	*H*
Chinese			J	P-H	P	F	H
Korean			J	P-H	P	F	H
Japanese				P-H	P		H
Pt. Hawaiian							H
Portuguese							H
Filipino							
Hawaiian							



GRAPH 4

MEAN SCORES IN ARITHMETIC REASONING FOR FEMALES

TABLE VIII

GROUPS (FEMALES) WHOSE MEANS IN ARITHMETIC REASONING ARE 3 P. E. OR MORE BELOW THOSE OF THE RACE IN THE LEFT HAND COLUMN

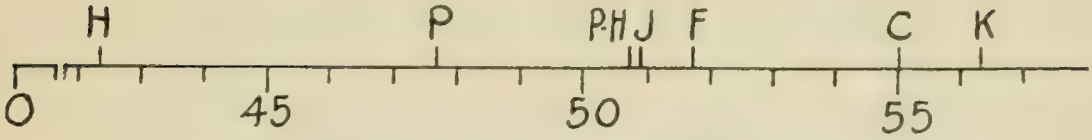
	K	*C*	*J*	*F*	*P-H*	*P*	*H*
Korean					P-H	P	H
Chinese			J		P-H	P	H
Japanese						P	H
Filipino							H
Pt. Hawaiian							H
Portuguese							H
Hawaiian							

TABLE IX

Differences between Means with the Probable Errors of Those Differences for Paragraph Meaning*

Males	Females			Jap.	Chi.	Port.	Haw.	P. Haw.	Kor.	Fil.
			M	*49.08*	*54.34*	*51.17*	*42.12*	*53.65*	*58.58*	*58.05*
			σ	*16.14*	*16.52*	*16.14*	*16.37*	*15.20*	*15 06*	*18.47*
	M	*σ*	*N*	*843*	*453*	*280*	*63*	*244*	*52*	*31*
Japanese	50.90	16.98	876	**1.82**	− 5.26	−2.09	6.96	− 4.57	− 9.50	− 8.97
				± .54	± .65	± .75	± 1.44	± .76	± 1.46	± 2.27
Chinese	55.05	17.13	442	−4.15	**.71**	3.17	12.22	.69	− 4.24	− 3.71
				± .67	**± .76**	± .84	± 1.48	± .84	± 1.50	± 2.64
Portuguese	47.61	16.56	139	3.29	7.44	**−3.56**	9.05	− 2.48	− 7.41	− 6.88
				±1.03	± 1.09	**±1.15**	± 1.54	± .92	± 1.55	± 2.33
Hawaiian	42.72	16.77	81	8.18	12.33	4.89	**.60**	−11.53	−16.46	−15.93
				±1.32	± 1.37	±1.57	**± 1.87**	± 1.54	± 1.98	± 2.64
Pt. Hawaiian	50.75	15.25	219	.15	4.30	−3.14	− 8.03	**− 2.90**	− 4.93	− 4.40
				± .80	± .88	±1.17	± 1.43	**± 1.16**	± 1.55	± 2.33
Korean	56.21	15.93	67	−5.31	− 1.16	−8.60	−13.49	− 5.46	**− 2.37**	.53
				±1.42	± 1.47	±1.66	± 1.85	± 1.55	**± 1.96**	± 2.64
Filipino	51.70	12.81	25	− .80	3.35	−4.09	− 8.98	− .95	4.51	**− 6.35**
				±1.77	± 1.81	±1.97	± 2.13	± 1.86	± 2.20	**± 2.83**

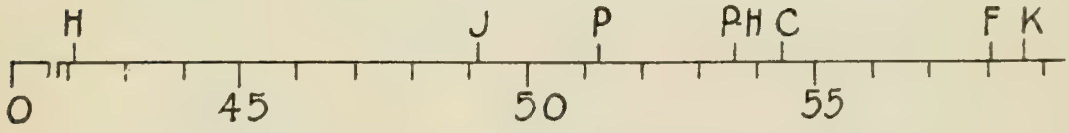
* Bold face figures in the diagonal are sex differences. Figures above and to the right of the diagonal give the stock differences for females and those below and to the left of the diagonal give the stock differences for the males. Differences for the males are obtained by subtracting from the mean of the race of a given column, successively, the values of the means of the races in the rows. Differences for the females are obtained by subtracting from the mean of the race of a given row successively the means of the races in the columns. The means, standard deviations, and populations are also given.



GRAPH 5
MEAN SCORES IN PARAGRAPH MEANING FOR MALES

TABLE X
GROUPS (MALES) WHOSE MEANS IN PARAGRAPH MEANING ARE 3 P. E. OR MORE BELOW THOSE OF THE RACE IN THE LEFT HAND COLUMN

	K	*C*	*F*	*J*	*P-H*	*P*	*H*
Korean				J	P-H	P	H
Chinese				J	P-H	P	H
Filipino							H
Japanese						P	H
Pt. Hawaiian							H
Portuguese							H
Hawaiian							



GRAPH 6
MEAN SCORES IN PARAGRAPH MEANING FOR FEMALES

TABLE XI
GROUPS (FEMALES) WHOSE MEANS IN PARAGRAPH MEANING ARE 3 P. E. OR MORE BELOW THOSE OF THE RACE IN THE LEFT HAND COLUMN

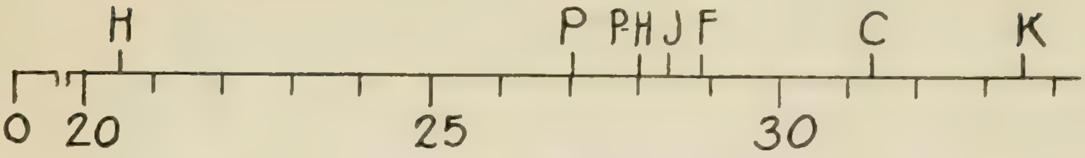
	K	*F*	*C*	*P-H*	*P*	*J*	*H*
Korean				P-H	P	J	H
Filipino						J	H
Chinese					P	J	H
Pt. Hawaiian						J	H
Portuguese							H
Japanese							H
Hawaiian							

TABLE XII

Differences between Means with the Probable Errors of Those Differences for Sentence Meaning*

Males	Females M	σ		*Jap.*	*Chi.*	*Port.*	*Haw.*	*P. Haw.*	*Kor.*	*Fil.*
			M	*27.03*	*30.79*	*28.06*	*22.13*	*30.58*	*31.96*	*30.53*
			σ	*13.49*	*14.15*	*13.46*	*11.95*	*12.69*	*13.18*	*13.74*
			N	*843*	*453*	*280*	*63*	*244*	*52*	*31*
Japanese	28.36	13.65	876	**1.33** **± .44**	−3.76 ± .55	− 1.03 ± .63	4.90 ± 1.06	− 3.55 ± .63	− 4.93 ± 1.27	− 3.50 ± 1.71
Chinese	31.28	14.26	442	−2.92 ± .55	**.49** **± .64**	2.73 ± .70	8.66 ± 1.11	.21 ± .71	− 1.17 ± 1.31	.26 ± 1.74
Portuguese	27.02	12.16	139	1.34 ± .76	4.26 ± .83	− **1.04** **± .88**	5.93 ± 1.15	− 2.52 ± .77	− 3.90 ± 1.35	− 2.47 ± 1.77
Hawaiian	20.51	12.53	81	7.85 ± .99	10.77 ±1.05	6.51 ± 1.17	− **1.62** **± 1.38**	− 8.45 ± 1.15	− 9.83 ± 1.60	− 8.40 ± 1.96
Pt. Hawaiian	28.02	13.14	219	.34 ± .67	3.26 ± .76	− 1.00 ± .92	− 7.51 ± 1.11	− **2.56** **± .81**	− 1.38 ± 1.35	.05 ± 1.77
Korean	33.50	14.35	67	−5.14 ±1.27	−2.22 ±1.31	− 6.48 ± 1.41	−12.98 ± 1.54	− 5.48 ± 1.37	**1.54** **± 1.74**	1.43 ± 2.08
Filipino	28.86	10.83	25	− .50 ±1.49	2.42 ±1.53	− 1.84 ± 1.62	− 8.35 ± 1.73	− .84 ± 1.58	4.64 ± 1.91	− **1.67** **± 2.23**

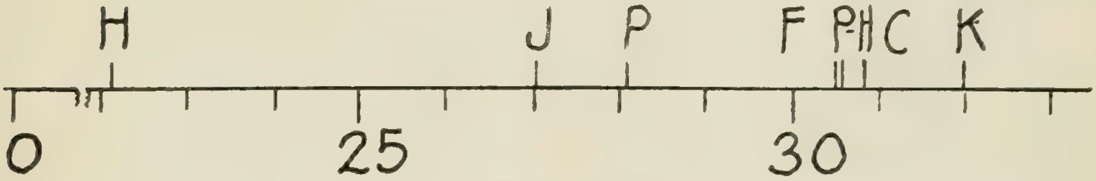
* Bold face figures in the diagonal are sex differences. Figures above and to the right of the diagonal give the stock differences for females and those below and to the left of the diagonal give the stock differences for the males. Differences for the males are obtained by subtracting from the mean of the race of a given column, successively, the values of the means of the races in the rows. Differences for the females are obtained by subtracting from the mean of the race of a given row successively the means of the races in the columns. The means, standard deviations, and populations are also given.



GRAPH 7
MEAN SCORES IN SENTENCE MEANING FOR MALES

TABLE XIII
GROUPS (MALE) WHOSE MEANS IN SENTENCE MEANING ARE 3 P. E. OR MORE BELOW THOSE OF THE RACE IN THE LEFT HAND COLUMN

	K	*C*	*F*	*J*	*P-H*	*P*	*H*
Korean				J	P-H	P	H
Chinese				J	P-H	P	H
Filipino							H
Japanese							H
Pt. Hawaiian							H
Portuguese							
Hawaiian							



GRAPH 8
MEAN SCORES IN SENTENCE MEANING FOR FEMALES

TABLE XIV
GROUPS (FEMALES) WHOSE MEANS IN SENTENCE MEANING ARE 3 P. E. OR MORE BELOW THOSE OF THE RACE IN THE LEFT HAND COLUMN

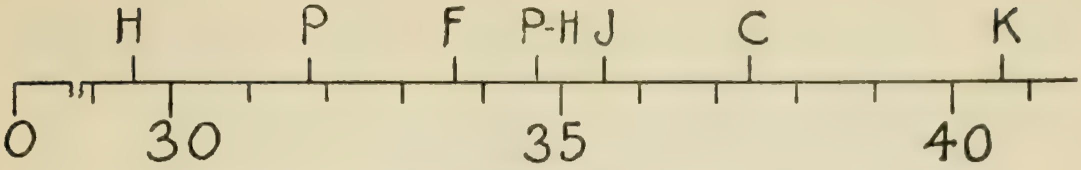
	K	*C*	*P-H*	*F*	*P*	*J*	H
Korean						J	H
Chinese					P	J	H
Pt. Hawaiian					P	J	H
Filipino							H
Portuguese							H
Japanese							H
Hawaiian							

TABLE XV

DIFFERENCES BETWEEN MEANS WITH THE PROBABLE ERRORS OF THOSE DIFFERENCES FOR WORD MEANING*

Males	*Females*			*Jap.*	*Chi.*	*Port.*	*Haw.*	*P. Haw.*	*Kor.*	*Fil.*
			M	*34.67*	*37.53*	*33.43*	*29.44*	*36.76*	*40.12*	*38.53*
			σ	*11.24*	*11 60*	*10.46*	*11.01*	*10.23*	*11.23*	*14.30*
	M	*σ*	*N*	*843*	*453*	*280*	*63*	*244*	*52*	*31*
Japanese	35.54	12.01	876	**.87** **± .38**	−2.86 ± .45	1.24 ± .50	5.23 ± .97	− 2.09 ± .51	− 5.45 ± 1.08	− 3.86 ± 1.77
Chinese	37.39	12.42	442	−1.85 ± .49	**− .14** **± .54**	4.10 ± .58	8.09 ± 1.01	.77 ± .57	− 2.59 ± 1.11	− 1.00 ± 1.79
Portuguese	31.83	12.33	139	3.71 ± .70	5.56 ± .76	**− 1.60** **± .77**	3.99 ± 1.03	− 3.33 ± .61	− 6.69 ± 1.13	− 5.10 ± 1.79
Hawaiian	29.45	11.39	81	6.09 ± .90	7.94 ± .94	2.38 ± 1.07	**.01** **± 1.27**	− 7.32 ± 1.03	−10.68 ± 1.41	− 9.09 ± 1.98
Pt. Hawaiian	34.71	11.52	219	.83 ± .59	2.68 ± .66	− 2.88 ± .84	− 5.26 ± 1.01	**− 2.05** **± .69**	− 3.36 ± 1.14	− 1.77 ± 1.80
Korean	40.66	11.94	67	−5.12 ±1.06	−3.27 ±1.10	− 8.83 ± 1.21	−11.21 ± 1.33	− 5.95 ± 1.15	**.54** **± 1.46**	1.59 ± 2.04
Filipino	33.66	8.87	25	1.88 ±1.23	3.73 ±1.26	− 1.83 ± 1.36	− 4.21 ± 1.47	1.05 ± 1.31	7.00 ± 1.57	**− 4.87** **± 2.12**

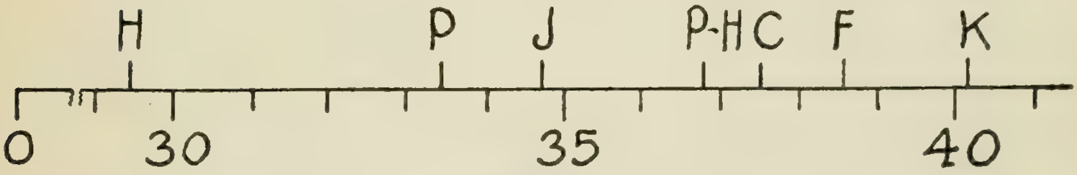
* Bold face figures in the diagonal are sex differences. Figures above and to the right of the diagonal give the stock differences for females and those below and to the left of the diagonal give the stock differences for the males. Differences for the males are obtained by subtracting from the mean of the race of a given column, successively, the values of the means of the races in the rows. Differences for the females are obtained by subtracting from the mean of the race of a given row successively the means of the races in the columns. The means, standard deviations, and populations are also given.



GRAPH 9
MEAN SCORES IN WORD MEANING FOR MALES

TABLE XVI
GROUPS (MALES) WHOSE MEANS IN WORD MEANING ARE 3 P. E. OR MORE BELOW THOSE OF THE RACE IN THE LEFT HAND COLUMN

	K	*C*	*J*	*P-H*	*F*	*P*	*H*
Korean		C	J	P-H	F	P	H
Chinese			J	P-H		P	H
Japanese						P	H
Pt. Hawaiian						P	H
Filipino							
Portuguese							
Hawaiian							



GRAPH 10
MEAN SCORES IN WORD MEANING FOR FEMALES

TABLE XVII
GROUPS (FEMALES) WHOSE MEANS IN WORD MEANING ARE 3 P. E. OR MORE BELOW THOSE OF THE RACE IN THE LEFT HAND COLUMN

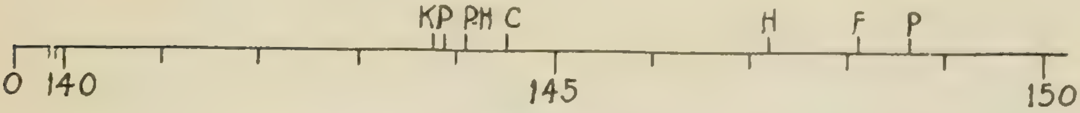
	K	*F*	*C*	*P-H*	*J*	*P*	*H*
Korean					J	P	H
Filipino							H
Chinese					J	P	H
Pt. Hawaiian					J	P	H
Japanese							H
Portuguese							H
Hawaiian							

TABLE XVIII

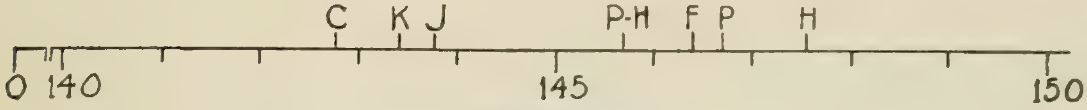
DIFFERENCES BETWEEN MEANS WITH THE PROBABLE ERRORS OF THOSE DIFFERENCES FOR AGE*

Males	*Females*			*Jap.*	*Chi.*	*Port.*	*Haw.*	*P. Haw.*	*Kor.*	*Fil.*
			M	*143.81*	*142.75*	*146.73*	*147.55*	*145.72*	*143.42*	*146.44*
			σ	*12.99*	*13.13*	*12.83*	*12.27*	*13.11*	*11.55*	*11.28*
	M	*σ*	*N*	*843*	*453*	*280*	*63*	*244*	*52*	*31*
Japanese	143.89	12.91	876	**.08** ± **.42**	1.06 ± .51	– 2.92 ± .60	– 3.74 ± 1.09	– 1.91 ± .64	.39 ± 1.12	– 2.63 ± .14
Chinese	144.51	13.38	442	– .62 ± .52	**1.76** ± **.60**	– 3.98 ± .66	– 4.80 ± 1.12	– 2.93 ± .70	– .67 ± 1.15	– 3.69 ± 1.44
Portuguese	148.66	11.99	139	–4.77 ± .74	–4.15 ± .88	**1.93** ± **.86**	– .92 ± 1.17	1.01 ± .77	3.31 ± 1.19	.29 ± 1.48
Hawaiian	147.19	12.59	81	–3.30 ± .99	–2.68 ±1.04	1.47 ± 1.17	– **.36** ± **1.41**	1.83 ± 1.19	4.13 ± 1.50	1.11 ± 1.73
Pt. Hawaiian	144.11	13.32	219	– .22 ± .67	.40 ± .74	4.55 ± .92	3.08 ± 1.12	– **1.61** ± **.83**	2.30 ± 1.22	– .72 ± 1.50
Korean	143.82	13.06	67	.07 ±1.15	.69 ± .80	4.84 ± 1.31	3.37 ± 1.46	.29 ± 1.27	**.40** ± **1.56**	3.02 ± 1.75
Filipino	148.10	13.41	25	–4.21 ±1.83	–3.59 ±1.86	.56 ± 1.94	– .91 ± 2.04	– 3.99 ± 1.91	– 4.28 ± 2.12	– **1.66** ± **2.28**

* Bold face figures in the diagonal are sex differences. Figures above and to the right of the diagonal give the stock differences for females and those below and to the left of the diagonal give the stock differences for the males. Differences for the males are obtained by subtracting from the mean of the race of a given column, successively, the values of the means of the races in the rows. Differences for the females are obtained by subtracting from the mean of the race of a given row successively the means of the races in the columns. The means, standard deviations, and populations are also given.



GRAPH 11
MEAN AGE FOR MALES



GRAPH 12
MEAN AGE FOR FEMALES

The mean scores on Arithmetic Computation for the Oriental groups are greater than those for the Portuguese and two Hawaiian groups. There is an interchange of positions between the males and females for the Hawaiian and Portuguese groups. This, however, is not significant in view of the other data.

Although a significant difference is found for the males between the Chinese and the Japanese, and the Chinese and Filipino males, no such difference is shown by the females.

Arithmetic Reasoning

Tables VI, VII, and VIII together with graphs 3 and 4 present the data and summary of the various groups. As in Arithmetic Computation, the Oriental group surpasses the Hawaiian and Portuguese groups. The Chinese, in three out of four comparisons made on the number tests, exceed the Japanese. The Part-Hawaiian group does better than the Hawaiian group in all four comparisons but only significantly so in the two comparisons on Arithmetic Reasoning.

Paragraph Meaning

Tables IX, X, and XI, in combination with graphs 5 and 6, give the data and a summary of the results on Paragraph Meaning. In a verbal test it would be expected that the Hawaiian group would do better than the others as a result of the English language being more generally used in the homes. All stocks, however, male and female, exceed the Hawaiians. The Koreans and Chinese here are considerably better than the Japanese, and they also surpass the Portuguese. The Part-Hawaiians do better than the Hawaiians.

Sentence Meaning

Tables XII, XIII, and XIV, in conjunction with graphs 7 and 8, present and summarize the material on Sentence Meaning.

The Koreans and Chinese again show a significant difference over the Japanese and Hawaiians, while the Part-Hawaiians surpass the Hawaiians.

Word Meaning

Tables XV, XVI, and XVII, together with graphs 9 and 10, give the material for Word Meaning. These tables corroborate the evidence presented for Paragraph and Sentence Meaning.

The Koreans and Chinese do better than the other stocks studied on all the verbal tests. Although the Korean group is relatively small ($N = 119$) the fact that in six comparisons they rank first in verbal material is of itself of considerable significance. Further evidence of a real difference is found in the number tests where in four comparisons they rank first once and second three times. Because each finding corroborates the others, more faith may be placed in the conclusions dealing with this group than would be justified by the size of the sample for a single comparison. As pointed out in Chapter II the Korean population came because of two motives. One of these motives may have caused the selection of a more intelligent strain. The Chinese are better in number and verbal material than the Japanese, Portuguese, and Hawaiians. They also exceed the Part-Hawaiians and in the case of the males, significantly so.

The Japanese excel the Part-Hawaiians, Portuguese, and Hawaiians in number material, but in verbal are only consistently better than the Hawaiians.

The Part-Hawaiians of both sexes are significantly better than the Hawaiians in Arithmetic Reasoning and in the three verbal tests. The only other difference found consistently for both sexes is that in Word Meaning where the Part-Hawaiians are superior to the Portuguese.

TABLE XVIII

THE NUMBER OF TIMES A STOCK IS SIGNIFICANTLY SUPERIOR TO OTHER STOCKS OF THE SAME SEX IN NUMBER AND VERBAL TESTS

Race	NUMBER TESTS Male	Female	Total	VERBAL TESTS Male	Female	Total	G.T.	Rank
Chinese	10	7	17	12	9	21	38	1
Korean	8	6	14	14	9	23	37	2
Japanese	6	5	11	5	3	8	19	3
Part-Hawaiian	2	2	4	4	8	12	16	4
Filipino	0	4	4	2	4	6	10	5
Portuguese	1	1	2	1	3	4	6	6
Hawaiian	0	0	0	0	0	0	0	7

Table XVIII shows the number of times a particular sex stock group significantly excels the other stocks of the same sex in number or verbal material. Roughly, the stocks are divided into three groups in number material, first, the Chinese and Koreans; second, the Japanese, Part-Hawaiians, and Filipinos; and third the Portuguese and Hawaiians.

In verbal material, as well as in the final ranking, the grouping suggested by Table XVIII is first, Chinese and Koreans; second, Japanese and Part-Hawaiians; and third, Filipinos, Portuguese, and Hawaiians.

Table XIX and graphs 11 and 12 show the ages of the various groups. Statistically certain stock age differences are not significant when dealing with small groups. An example of this is the difference of 4.28 months between the Korean and Filipino males. It is true that psychologically an age difference may be significant and important, while it may fail to meet the requirements for statistical significance.

The intercorrelations between the various variables are given in Tables XX to XXVI inclusive. (See Appendix I.) Each table gives the information for both males and females of a particular stock and in addition to the intercorrelations gives the means and standard deviations. The correlation over a four year range between age and its square is given for the fourteen groups and ranges between the values .99859 and .99773.*

In Table XX for the Filipino males the coefficient of correlation between Arithmetic Computation and Arithmetic Reasoning is less than that between Arithmetic Computation and any of the verbal tests; in fact it is lower than the correlation between Arithmetic Computation and either of the two age variables. Such a low correlation can be expected by chance from the 56 intercorrelations with Arithmetic Computation, particularly since N for the Filipino males is only 25.

---

* This is so nearly perfect that consideration of the square of age as a variable over a short age range seems unnecessary, although the square of age has been partialed out in this study.

Tables XXVII to XXXIII inclusive give the second order correlation coefficients resulting when age and square of age have been partialed out.* Each table gives the data for the males and females of a particular race. These coefficients are the basis of the tetrads given in Tables XXXIV and XXXV.

If two stocks have the same mean mental ability, but have different variabilities, the more variable stock will produce more men of great superiority as well as more men of great inferiority. Since the progress of a stock depends to a considerable degree on the superior persons in that stock, the more variable stock will tend to advance beyond the less variable stock. It should be noted that in this investigation there are no significant differences in variability between the stocks on the various subtests.

RESUME OF RESULTS

1. In the particular stocks studied the mean scores in number material for the three Oriental groups (Chinese, Korean, and Japanese) were greater than those for the Part-Hawaiian, Filipino, Portuguese, and Hawaiian groups.

2. The Koreans and Chinese in verbal material do better than the Part-Hawaiians, Japanese, Filipinos, Portuguese, and Hawaiians.

3. On the basis of the means of all the tests it is possible to divide the stocks into three distinct groups, first, Chinese and Koreans; second, Japanese and Part-Hawaiians, and third, Filipinos, Portuguese, and Hawaiians.

---

* As a rough check on the technique of partialing out age and its square as against controlling age by experimental methods, the inter-correlations between the number and verbal tests were calculated for 104 Japanese males age 12-0 to 12-3. The average intercorrelation for this subgroup was .61+. The average intercorrelation for all Japanese males as given in Table XX is .66−, and for the same group when age and its square have been removed, Table XXVII gives .58+. Obviously by their nature these coefficients are not strictly comparable, yet they are sufficiently so to indicate that for our purpose no serious statistical error has been introduced by partialing out the two age variables.

4. The Hawaiians consistently rank lower than any other stock in both verbal and number material.

5. The Part-Hawaiians rank fourth in number material, third in verbal material, and fourth when number and verbal are combined.

CHAPTER IV

The Organization of Abilities in the Six Stocks

The measurements used in Chapter III to discover differences in mean ability and in variability between stocks can be used to test certain general hypotheses concerning mental organization, especially (a) that ability in a composite of ordinary intelligence tests or achievement tests in matters depending on intelligence, consists of one general ability plus other abilities all of which are specific, and (b) that, on the contrary, some of the latter overlap to form group factors.

The data available here are most advantageous for such an investigation. The scores on the several subtests are highly reliable; the variability is approximately the same from group to group; some of the samples are very large in comparison with those which have been used previously in such arguments; and heterogeneity has been avoided or reduced more than hitherto. Most important of all, the groups represent samples of populations clearly distinct as to ancestry, so that if they agree in displaying a group factor or in not displaying one, the argument is very strong.

For the convenience of the reader the general procedure of the tetrad technique which is to be adapted to this investigation will be briefly reviewed.*

The tetrad technique permits a mathematical demonstration that certain assumptions made regarding the traits involved in test scores are not untenable. For instance, it might be assumed that a child's score on a particular test depends upon his ability to deal with verbal relations, and some traits specific to the test. To prove that such an assumption is tenable requires first, more information and, second, specialized treatment. At least four measures of this function are required before this hypothesis can be tested.

---

* Professor Spearman originally developed the tetrad difference technique.

Let

$$\begin{aligned} x_1 &= c_1a+e_1 \\ x_2 &= c_2a+e_2 \\ x_3 &= c_3a+e_3 \\ x_4 &= c_4a+e_4 \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where x is a derivation from the mean of a given test.*

Now the correlation between variables 1 and 2 is found by multiplying together the first two equalities in (1), summing and dividing by the total frequency of the sample so that

$$r_{12} = (c_1c_2\sigma_a^2)/\sigma_1\sigma_2$$

since $c_1\Sigma ae_2$, $c_2\Sigma ae_1$, and Σe_1e_2 equal zero.

Define

$$\begin{aligned} \alpha_1 &= c_1\sigma_a/\sigma_1 \qquad & \alpha_3 &= c_3\sigma_a/\sigma_3 \\ \alpha_2 &= c_2\sigma_a/\sigma_2 \qquad & \alpha_4 &= c_4\sigma_a/\sigma_4 \end{aligned}$$

Then

$$\begin{aligned} r_{12} &= \alpha_1\alpha_2 \qquad & r_{24} &= \alpha_2\alpha_4 \\ r_{13} &= \alpha_1\alpha_3 \qquad & r_{24} &= \alpha_2\alpha_4 \\ r_{14} &= \alpha_1\alpha_4 \qquad & r_{34} &= \alpha_3\alpha_4 \end{aligned}$$

If equations (1) hold then certain relationships will hold among the correlation coefficients so that proposition I may be stated as follows:

Proposition I: Four variables may be thought of as due to one general factor plus four specific factors when $r_{12}r_{34} = r_{13}r_{24} = r_{14}r_{23} = \alpha_1\alpha_2\alpha_3\alpha_4$.

From these relations

$$\begin{aligned} r_{12}r_{34} - r_{13}r_{24} &= 0 \\ r_{12}r_{34} - r_{14}r_{23} &= 0 \\ r_{13}r_{24} - r_{14}r_{23} &= 0 \\ r_{13}r_{24} - r_{12}r_{34} &= 0 \\ r_{14}r_{23} - r_{12}r_{34} &= 0 \\ r_{14}r_{23} - r_{13}r_{24} &= 0 \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

---

* c_1, c_2, c_3, c_4, are constants for particular tests; "a" is the general factor, and e_1, e_2, e_3, and e_4 are factors specific to particular tests.

Only the first two equalities are independent. The equations in (2) have been termed tetrad differences by Spearman (1927) who first derived them. Now, for convenience designate the first equality in (2), t_{1234}, the second, t_{1243}, and the third, t_{1342}.

If the data meet the criteria of (2) it may be assumed that the hypothesis of one general factor and four specific factors is tenable. If the data do not, a slightly more complex pattern must be hypothesized and tested.

Let

$$\begin{aligned} x_1 &= c_1 a + k_1 b + e_1 \\ x_2 &= c_2 a + k_2 b + e_2 \\ x_3 &= c_3 a \qquad\;\; + e_3 \\ x_4 &= c_4 a \qquad\;\; + e_4 \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where a, b, and e are uncorrelated with each other.

Now

$$x_1 = c_1 a + k_1 b + e_1$$

so that

$$\sigma_1^2 = c_1^2 \sigma_a^2 + k_1^2 \sigma_b^2 + \sigma_{e_1}^2$$

Define

$$\beta_1 = k_1 \sigma_b / \sigma_1$$

and similarly

$$\beta_2 = k_2 \sigma_b / \sigma_2$$

Multiplying together the first two equations in (3), summing and dividing by the total frequency of the sample gives

$$r_{12} = (c_1 c_2 \sigma_a^2)/(\sigma_1 \sigma_2) + (k_1 k_2 \sigma_b{}^2)/(\sigma_1 \sigma_2)$$

The other correlation coefficients may be determined in a similar manner. Applying the definitions of α's and β's gives

$$\begin{aligned} r_{12} &= \alpha_1 \alpha_2 + \beta_1 \beta_2 & r_{23} &= \alpha_2 \alpha_3 \\ r_{13} &= \alpha_1 \alpha_3 & r_{24} &= \alpha_2 \alpha_4 \\ r_{14} &= \alpha_1 \alpha_4 & r_{34} &= \alpha_3 \alpha_4 \end{aligned}$$

The relationships of these correlation coefficients may be expressed as follows:

Proposition II: Four variables may be thought of as due to one general factor, one group factor common to two of the variables, and four specific factors when

$$t_{1234} = r_{12}r_{34} - r_{13}r_{24} \neq 0$$
$$t_{1243} = r_{12}r_{34} - r_{14}r_{23} \neq 0$$
$$t_{1342} = r_{13}r_{24} - r_{14}r_{23} = 0$$

and

$$t_{1234} = t_{1243}$$

If the data meet these conditions it may be assumed that the hypothesis of one general and one group factor common to two is tenable.

This study applies the tetrad analysis to five variables. An inspection of the tests will show that all contain verbal material in varying degrees. The Arithmetic Computation and Arithmetic Reasoning tests call into use other types of activities than verbal. Most workers in the theory of factors recognize the verbal factor, and the work of Kelley (1928), Schneck (1929), and of many others point to the existence of a number factor.

Let

X_1 = Arithmetic Computation
X_2 = Arithmetic Reasoning
X_3 = Paragraph Meaning
X_4 = Sentence Meaning
X_5 = Word Meaning

and

$$\begin{aligned} x_1 &= c_1a + k_1b + e_1 \\ x_2 &= c_2a + k_2b + e_2 \\ x_3 &= c_3a \qquad\quad + e_3 \\ x_4 &= c_4a \qquad\quad + e_4 \\ x_5 &= c_5a \qquad\quad + e_5 \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots (4)$$

where x is a deviation from the mean.*†

---

* c_1 is a constant not changing from individual to individual in a group, when considering x_1, similarly, the other c's and the k's. e_1, e_2, e_3, e_4, and e_5 represent specific factors and so are uncorrelated with each other or with a and b. The general factor, a, is common to all the tests, and here is hypothesized verbal. The group factor, b, is common to x_1 and x_2 and here is hypothesized to be numerical ability. After an inspection of the tests these assumptions were made.

† The evidence and arguments which follow, however, would be the same in every particular, if what has just been called "verbal" were called V, or general factor, or general intelligence, or X or K, and if what has just been designated "number" were called N, or group factor, or mathematical intelligence, or P or D.

If the criteria of Proposition II are met by the data of this study the hypothesis of a general factor and a group factor will be tenable.

Further, if the pattern on page 35 is tenable the independent tetrad differences can be defined as follows:

$$
\begin{aligned}
t_{1234} &= (\alpha_1\alpha_2+\beta_1\beta_2)\alpha_3\alpha_4-\alpha_1\alpha_3\alpha_2\alpha_4 \neq 0\\
t_{1243} &= (\alpha_1\alpha_2+\beta_1\beta_2)\alpha_3\alpha_4-\alpha_1\alpha_4\alpha_2\alpha_3 \neq 0\\
t_{1235} &= (\alpha_1\alpha_2+\beta_1\beta_2)\alpha_3\alpha_5-\alpha_1\alpha_3\alpha_2\alpha_5 \neq 0\\
t_{1253} &= (\alpha_1\alpha_2+\beta_1\beta_2)\alpha_3\alpha_5-\alpha_1\alpha_5\alpha_2\alpha_3 \neq 0\\
t_{1245} &= (\alpha_1\alpha_2+\beta_1\beta_2)\alpha_4\alpha_5-\alpha_1\alpha_4\alpha_2\alpha_5 \neq 0\\
t_{1254} &= (\alpha_1\alpha_2+\beta_1\beta_2)\alpha_4\alpha_5-\alpha_1\alpha_5\alpha_2\alpha_4 \neq 0\\
t_{1345} &= \alpha_1\alpha_3\alpha_4\alpha_5-\alpha_1\alpha_4\alpha_3\alpha_5 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5)\\
t_{1354} &= \alpha_1\alpha_3\alpha_4\alpha_5-\alpha_1\alpha_5\alpha_3\alpha_4 = 0\\
t_{2345} &= \alpha_2\alpha_3\alpha_4\alpha_5-\alpha_2\alpha_4\alpha_3\alpha_5 = 0\\
t_{2354} &= \alpha_2\alpha_3\alpha_5\alpha_4-\alpha_2\alpha_5\alpha_3\alpha_4 = 0
\end{aligned}
$$

As pointed out above only two of each group of three tetrad differences are independent, so henceforth only the independent tetrad differences are considered. If the hypothesis as to the distribution of the group factor is correct tetrad differences t_{1345}, t_{1354}, t_{2345}, and t_{2354} should equal zero within their sampling error. Further, tetrad differences t_{1234}, t_{1243}, t_{1235}, t_{1253}, t_{1245}, and t_{1254} should not equal zero within their sampling error.

It is necessary to know the standard error of a tetrad difference. Two such formulas have been offered, one by Spearman and Holzinger (1924) and a more general formula by Pearson and Moul (1927). The standard error of a tetrad difference as given by Pearson and Moul is

$$
\begin{aligned}
\sigma_{t_{1234}} = (1/N^{1/2})[&r_{12}^2+r_{13}^2+r_{24}^2+r_{34}^2+2r_{12}r_{14}r_{23}r_{34}\\
&+2r_{13}r_{14}r_{23}r_{24}-2r_{12}r_{13}r_{23}-2r_{12}r_{14}r_{24}\\
&-2r_{13}r_{14}r_{34}-2r_{23}r_{24}r_{34}\\
&+t_{1234}^2(r_{12}^2+r_{13}^2+r_{14}^2+r_{23}^2+r_{24}^2+r_{34}^2-4)]^{1/2}
\end{aligned}
\qquad \ldots\ldots\ldots\ldots\ldots\ldots (6)
$$

The mean of a set of tetrad differences of the nature t_{12ij} is a more stable measure than any particular tetrad, and accordingly was used. This value also is a measure of the bond strength of x_1 and x_2. Before such a function can be profitably employed it is necessary to determine its standard error. The standard error of the mean tetrad difference t_{12ij} is

$$\begin{aligned}\sigma^2_{t_{12}} = {} & [1/(n-2)(n-3)][2r_{12}^2\sigma^2+2(n-3)r_1^2\sigma^2+(n-2)(n-3)r^2\sigma^2 \\ & +4(n-4)r_{12}^2q+(n-4)(n-5)r_{12}^2p-8(n-3)r_{12}r_1q \\ & -4(n-3)(n-4)r_{12}r_1p+2(n-2)(n-3)r_{12}rp \\ & +2(n-2)(n-3)r_1^2q+2(n-3)^2r_1^2p-4(n-2)(n-3)r_1rq] \qquad \ldots\ldots\ldots\ldots\ldots\ldots (7)\end{aligned}$$

This formula supersedes the approximation given by Kelley (1928) and should be used in its stead.*

The derivation of this formula and the particular meanings to the symbols r_{12},r_1, r, p, q, n, and N are given in Appendix II.

The tetrad difference assumes linearity among the variables. Over a four year age range there is for the tests considered a slight curvilinear correlation with age. If the progressive change could be represented effectively by a straight line it would be sufficient to consider age as another variable and to eliminate it by calculating the corresponding partial correlation coefficient. The equation, however, is not so simple, but roughly is of the type

$$X=A+bx+cx^2$$

The partial correlation required is one found by eliminating x and x^2, i.e., age and the square of age.

---

* Dr. Kelley in a personal communication, regarding the two formulas, and the effect on the findings in "Crossroads in the Mind of Man," states, "My derivation makes certain substitutions of mean correlations of the sort $r_{r_{ij}r_{ik}}$ for mean correlations of the sort $r_{r_{ij}r_{kh}}$ and the reverse. The interchanges just about balance, for using our common notation and letting $\delta=\bar{r}_{r_{ij}r_{ik}} - r_{r_{ij}r_{kh}}$, I find that my incorrect formula and yours would be equal in the case of the six variable problem, in case the following expression were zero:

$$\sigma^2[\ \{\delta(6r_{12}^2+2r_1^2-8r_1r_{12})\}\ +\{2r_{12}^2(1-\bar{r}_{r_{12}r_{34}})-4r_1r_{12}\delta\}]$$

Both { } terms must, with such correlations as appear in *Crossroads*, be very small. They are, however, presumably both positive, so that my calculated standard errors may be a trifle large. I am glad the error was in this direction, as, if material, as I am sure it was not, it would tend to favor the single general factor interpretation of results, rather than the interpretation which I find clearly evident, namely that of several independent mental factors.

"I would deem it a favor if you would see fit to announce my withdrawal of formula (64)."

It is interesting to note that the more accurate formula tends to strengthen Dr. Kelley's line of argument.

Fisher (1925) has shown that the true sampling distribution of partial correlation coefficients calculated on a group of N individuals is the same as that of a zero order coefficient determined from a sample of $N-n$ individuals, where n is the number of variables eliminated. With samples as large as those of the Japanese, Chinese, Portuguese and Part-Hawaiian groups used here no serious error is introduced in using the constants of the sample. A slight error is introduced in the Hawaiian and Korean groups, and an appreciable error in the Filipino group. Partial correlation is used to secure linearity of the variables and homogeneity with respect to age. The partial correlations were used as a basis for the computation of the tetrad differences.

Tetrad differences of the nature t_{1ijk} and t_{2ijk} should equal zero within the limits of their errors if the following hypothesized pattern is tenable.

$$\begin{aligned} x_1 &= c_1a + k_1b + e_1 \\ x_2 &= c_2a + k_2b + e_2 \\ x_3 &= c_3a \qquad\quad + e_3 \\ x_4 &= c_4a \qquad\quad + e_4 \\ x_5 &= c_5a \qquad\quad + e_5 \end{aligned}$$

where variables 1 and 2 refer to number tests and variables $i=3$, $j=4$, and $k=5$ to verbal tests.

Table XXXIV presents the tetrad differences of the type t_{1ijk} and t_{2ijk} together with the probable error of the largest tetrad difference. These probable errors were calculated by formula (7). The four independent tetrad differences t_{1345}, t_{1354}, t_{2345}, and t_{2354} are by hypothesis free of the number bond and should equal zero within their errors. The largest tetrad difference for each stock was selected and its probable error calculated.*

---

* This calculated probable error of the largest tetrad difference, however, cannot be treated as an ordinary error for the largest of four was deliberately chosen. To infer the probability of its being significantly different from zero by means of the table of probabilities derived from the normal probability curve the tetrad difference should have been selected at random. It is assumed in this study that the tetrad difference is distributed normally, for the exact distribution is not known to the writer.

The criterion that a measure will lie outside a given range once in twenty times will be used, following Fisher (1925). The point of dichotomy is approximately two sigma. Accepting this criterion it is necessary to determine a new point of dichotomy, so that the chances that the largest of four independent measures will lie outside this range are one in twenty.

Let k equal the area to the point of dichotomy, at which the chances are nineteen in twenty that the largest of four independent measures will be included.

In Table XXXIV only three tetrad differences out of fifty-six are significantly different from zero. One of these, t_{2354} for the Filipino males can be explained by the unusually large error of measurement in the original correlations.

The tetrad difference t_{1354} for both the Chinese males and females is significantly different from zero. For these two groups the hypothesis is not tenable that Arithmetic Computation, Paragraph Meaning, Sentence Meaning, and Word Meaning have only a general factor and four specific factors. That this should happen in both sexes may probably be of considerable significance. Data for the proof of any other factor pattern is unavailable for this stock in this study. The significant difference from zero of the tetrad difference t_{1354} may indicate the presence of a group factor for the Chinese which is not present in the other stocks.

Table XXXV gives the independent tetrad differences in which a number-verbal bond may appear, for each stock and sex group. The last column gives the mean tetrad difference with its probable error. When these values are significantly different from zero, it is probable that a number bond is present.

The Hawaiian males, the Filipino females and both sexes of the Japanese, Chinese, Portuguese, Part-Hawaiians, and Koreans have a strong number factor present. (See Table XXXV.)

The Hawaiian females just fail to meet the criterion of significance of three times the probable error. From consideration of the other data this insufficiency does not invalidate the conclusion that a number factor is present.

The unreliability of all the intercorrelations for the Filipino males, and especially of r_{12}, makes any inferences from tetrad differences for this group inconclusive. The data for this stock, hereafter, are presented, though inferences arrived at neither stand nor fall upon the data of this stock.

Then

$\pm k^4 = .9500$

$\pm k = .9873$

$+ k = .4936$

.4936% of the area is included in 2.490192σ.

Thus the largest of four tetrad differences may lie within ±2.49 sigma and still not be significantly different from zero. When the random tetrad difference is selected, the chances are nineteen in twenty that it lies within the range of ± 2 sigma, when, however, the largest tetrad difference of four is selected the chances are nineteen in twenty that it lies within the range of ± 2.49 sigma.

TABLE XXXIV

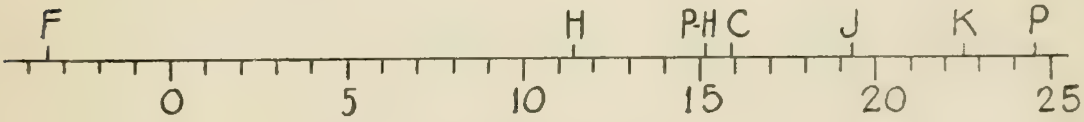
Tetrads Involving One Number and Three Verbal Tests, Together with the Standard Error of the Largest Tetrad in Each Group. Starred Tetrads Are Significantly Different from Zero

Race	t_{1345}	t_{1354}	t_{2345}	t_{2354}
Japanese				
Male	.0111	.0214	.0159	.0333 ±.0170
Female	.0188	.0214	.0326	.0391 ±.0147
Chinese				
Male	.0140	.0643* ±.0164	−.0128	.0418
Female	−.0119	.0812* ±.0196	.0029	.0283
Portuguese				
Male	−.0038	.0484 ±.0339	−.0065	.0434
Female	−.0493 ± 0259	−.0097	−.0086	.0322
Hawaiian				
Male	−.0294	.0491	−.0894 ±.0505	−.0339
Female	.0229	−.0603	.0009	.0613 ±.0548
Pt. Hawaiian				
Male	−.0027	.0051	.0225	.0373 ±.0282
Female	.0269	.0066	.0645 ±.0422	.0286
Korean				
Male	.0906 ±.0610	.0063	.0293	.0621
Female	−.0106	.1162 ±.0601	.0545	−.0087
Filipino				
Male	.1343	.1927	.1184	.2053* ±.0527
Female	−.1150	.0022	−.1708 ±.0994	.0359

It should be kept in mind that the tetrad differences are a relative measure of the ratio of number and verbal strength. That the tetrad difference gives only the relative strength of two factors and not the absolute difference must be kept clearly in mind. The number strength of stock A may be significantly greater than that of stock B, whereas the absolute scores on a number test may place stock B above stock A. The fact that the number-verbal ratio is greater for stock A than for stock B, tells nothing as to the absolute rank of the two stocks on a test.

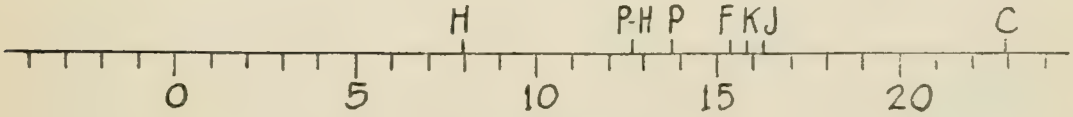
Stock idiosyncrasy differences are differences between the proportional strengths of the number factors of two stocks. Sex idiosyncrasy differences refer to differences between the proportional strengths of the two number factors for the males and females of a single stock.

Table XXXVI and graphs 13 and 14 show the tetrad differences and the significance of their differences. Two groups, the Portuguese males and Chinese females, show distinct differences in the relative strength of number versus verbal ability to that apparent in certain of the other groups. The Portuguese males differ significantly from the Chinese, Part-Hawaiian, and Hawaiian males. Although the Portuguese males have relatively a greater number ability, the Chinese and Part-Hawaiian males do better in absolute test scores, while the Hawaiian males have less absolute as well as a less relative number ability than the Portuguese males. The Chinese females have a significantly greater number-verbal ratio than the Portuguese, Part-Hawaiian, and Hawaiian females. The Chinese females not only exceed in relative ability but also in absolute ability on the number tests.



GRAPH 13

MEAN TETRADS, t_{11ij}, FOR MALES



GRAPH 14

MEAN TETRADS, t_{12ij}, FOR FEMALES

In general it is fair to conclude that there is a number factor present for all stocks.* The tetrad technique offers a means for showing differences between stocks in their relative number-verbal abilities. This may or may not agree with their ranking on a scale of absolute scores of a test designed to measure that particular trait.

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* Except for the Filipino males. See page 39.

TABLE XXXV

MEAN NUMBER VERBAL TETRADS WITH THEIR P.E.'S AND THE BASIC TETRADS FOR THE VARIOUS RACIAL GROUPS

Race	t_{1234}	t_{1243}	t_{1235}	t_{1253}	t_{1245}	t_{1254}	t_{12}
Japanese, M.	.1711	.1697	.2135	.2084	.1974	.1945	.1924 ±.0107
Japanese, F.	.1514	.1444	.1715	.1613	.1730	.1707	.1621 ±.0091
Chinese, M.	.1104	.1319	1688	.1940	.1716	.1752	.1587 ±.0073
Chinese, F.	.1971	.1862	.2307	.2735	.2182	.2708	.2294 ±.0126
Portuguese, M.	.2114	.2128	.2700	.2789	.2450	.2520	.2450 ±.0219
Portuguese, F.	.1359	.1019	.1815	.1439	.1316	.1274	.1370 ±.0159
Hawaiian, M.	.0667	.1058	.1167	.1830	.0879	.1200	.1134 ±.0254
Hawaiian, F.	.0305	.0502	.0906	.0914	.1155	.0968	.0792 ±.0281
Pt. Hawaiian, M.	.1498	.1306	.1713	.1452	.1577	.1520	.1511 ±.0143
Pt. Hawaiian, F.	.1374	.1135	.1220	.1055	.1360	.1457	.1267 ±.0153
Korean, M.	.1770	.2309	.2507	.2006	.2903	.1984	.2247 ±.0341
Korean, F.	.1644	.1118	.1161	.2285	.0950	.2327	.1581 ±.0366
Filipino, M.	−.1291	−.0853	−.0539	−.0188	.0398	.0393	−.0347 ±.0498
Filipino, F.	.0596	.0956	.2820	.2500	.1571	.0769	.1535 ±.0484

TABLE XXXVI

DIFFERENCES BETWEEN VARIOUS MEAN TETRADS, t_{12}, WITH THEIR PROBABLE ERRORS*

	t_{12}	$\frac{t_{12}}{\sigma}$	*Jap.* *.1621* *.0135*	*Chi.* *.2294* *.0187*	*Port.* *.1370* *.0235*	*Haw.* *.0792* *.0416*	*P. Haw.* *.1267* *.0227*	*Kor.* *.1581* *.0542*	*Fil.* *.1535* *.0718*
Japanese	.1924	.0158	**.0303** ±**.0139**	−.0673 ±.0155	.0251 ±.0183	.0829 ±.0295	.0354 ±.0178	.0040 ±.0376	.0086 ±.0492
Chinese	.1587	.0108	.0337 ±.0129	−**.0707** ±**.0146**	.0924 ±.0202	.1502 ±.0308	.0783 ±.0193	.0713 ±.0386	.0759 ±.0500
Portuguese	.2450	.0324	−.0526 ±.0244	−.0863 ±.0231	**.1080** ±**.0270**	.0578 ±.0322	.0103 ±.0220	−.0211 ±.0398	−.0165 ±.0509
Hawaiian	.1134	.0376	.0790 ±.0277	.0453 ±.0265	.1316 ±.0334	**.0342** ±**.0380**	−.0640 ±.0320	−.0789 ±.0461	−.0743 ±.0560
Pt. Hawaiian	.1511	.0212	.0413 ±.0179	.0076 ±.0160	.0939 ±.0261	−.0377 ±.0519	**.0244** ±**.0210**	− 0314 ±.0396	−.0268 ±.0508
Korean	.2247	.0505	−.0323 ±.0363	−.0660 ±.0348	.0203 ±.0405	−.1113 ±.0427	−.0736 ±.0370	**.0666** ±**.0499**	+.0046 ±.0606
Filipino	−.0347	.0739	.2271 ±.0509	.1934 ±.0504	.2797 ±.0544	.1481 ±.0560	.1858 ±.0519	.2594 ±.0604	**.1882** ±**.0695**

---

* Bold face figures in the diagonal are sex differences. Figures above and to the right of the diagonal give the differences for females, and those below and to the left give differences for the males. Differences for the males are obtained by subtracting from the t_{12} value of a race of a given column, successively the t_{12} values of the races in the rows. Differences for the females are obtained by subtracting from the t_{12} value of a race of a given row, successively the t_{12} values of the races in the columns.

The Tables XXXIV and XXXV demonstrate abundantly that in every group the data satisfy the criteria for one general factor, one group factor common to two variables and four specific factors. The existence of a group factor for the tests and groups used then seems to be beyond question.

CHAPTER V

THE IDIOSYNCRASIES OF STOCKS IN THE COMPARATIVE MAGNITUDES OF NUMERICAL AND VERBAL ABILITIES

Since the data satisfies the pattern hypothesized on page 38, it is possible to ascribe proportionate weights to $\alpha_1\alpha_2$ and $\beta_1\beta_2$. As the following analysis will be in terms of the variances, it is permissible to express the strength of the numerical factor as a per cent of the verbal factor.

Proposition III: Given four variables with a general factor common to four, and a group factor common to two of them, it is possible to ascribe numerical values to the common factor in each of the four variables.

Defining α_1, α_2, α_3, α_4, α_5, β_1, β_2, and x_1, x_2, x_3, x_4, and x_5 as before, then the following equations may be written:

$$r_{12} = \alpha_1\alpha_2 + \beta_1\beta_2 \qquad r_{23} = \alpha_2\alpha_3$$

$$r_{13} = \alpha_1\alpha_3 \qquad r_{24} = \alpha_2\alpha_4$$

$$r_{14} = \alpha_1\alpha_4 \qquad r_{34} = \alpha_3\alpha_4$$

$$\begin{aligned} \alpha_1^2 &= r_{13}r_{14}/r_{34} \\ \alpha_2^2 &= r_{23}r_{24}/r_{34} \\ \alpha_3^2 &= r_{13}r_{34}/r_{14} \\ \alpha_4^2 &= r_{14}r_{34}/r_{13} \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

It is necessary to have the standard error of such functions and this has been given by Cureton (1931).*

Proposition IV: Given five variables with a general factor common to all, and a group factor contained in two variables, it is always possible to analyze the covariance into two addible portions.

This may be done in either of two ways, but the average value resulting from the methods would seem most appropriate.

---

* $$\sigma_F^2 = [(r_{12}r_{13})^2/(r_{23}^2 N)][1/r_{12}^2 + 1/r_{13}^2 + 1/r_{23}^2 + 4r_{12}r_{13}/r_{23} + 2r_{23}/r_{12}r_{13} - 2r_{13}/r_{12}r_{23} - 2r_{12}/r_{13}r_{23} - 5]$$

where $F = r_{12}r_{13}/r_{23}$

Now

$$\beta_1\beta_2 = r_{12} - \alpha_1\alpha_2$$

$$\beta_1\beta_2 = r_{12} - (r_{13}r_{14}r_{23}r_{24})^{1/2}/r_{34} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

as a check on this value, remembering that

$$t_{1234} = \alpha_1\alpha_2\alpha_3\alpha_4 + \alpha_3\alpha_4\beta_1\beta_2 - \alpha_1\alpha_3\alpha_2\alpha_4 = \alpha_3\alpha_4\beta_1\beta_2$$

$$\beta_1\beta_2 = t_{1234}/r_{34} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

This study is concerned with the number verbal ratio, $\beta_1\beta_2/\alpha_1\alpha_2$, in the different stocks. This ratio can be determined as follows:

$$\beta_1\beta_2/\alpha_1\alpha_2 = t_{1234}/(r_{13}r_{14}r_{23}r_{24}()^{1/2}$$

$$= (r_{12}r_{34} - r_{13}r_{24})/(r_{13}r_{14}r_{23}r_{24})^{1/2} \ldots\ldots\ldots\ldots\ldots (11)$$

Now designate as the *tetrad ratio*

$$t_{12-34} = r_{12}r_{34}/r_{13}r_{24},\ t_{12-43} = r_{12}r_{34}/r_{14}r_{23},\ \text{etc.}$$

So that,

$$\beta_1\beta_2/\alpha_1\alpha_2 = (t_{12-34}t_{12-43})^{1/2} - (t_{13-42})^{1/2} \qquad \ldots\ldots\ldots\ldots\ldots (12)$$

This gives the ratio in the form adapted for calculation, and in a form from which it is possible to determine its standard error.*

$$F = (t_{12-34}t_{12-43})^{1/2} - (t_{13-42})^{1/2}$$

$$\text{Let } a = t_{12-34}$$

$$b = t_{12-43}$$

$$c = t_{13-42}$$

$$dF = (adb + bda)/2(ab)^{1/2} - dc/2c^{1/2}$$

Squaring, summing, and dividing by the number of samples gives

$$\sigma_F^2 = [1/4Nabc][a^2c\sigma_b^2 + b^2c\sigma_a^2 + ab\sigma_c^2 + 2abcp_{ab}$$
$$- 2a(abc)^{1/2}p_{bc} - 2b(abc)^{1/2}p_{ac}] \ldots\ldots\ldots\ldots (13)$$

To evaluate this expression, the variance of a tetrad ratio and the convariance between two tertad ratios are needed. The variance of a tetrad ratio is

---

* The exact distribution of the tetrad ratio is not known. As a first approximation a normal distribution may be assumed and it is with this assumption that the standard errors have been interpreted.

$$\begin{aligned}\sigma^2_{12-34} = [t^2_{12-34}/N][&1/r^2_{12}+1/r^2_{13}+1/r^2_{24}+1/r^2_{34}+2/t_{12-34}\\ &+2t_{13-42}+2/t_{12-43}+2/t_{13-42}\\ &-2r_{23}/r_{12}r_{13}-2r_{14}/r_{12}r_{24}-2r_{14}/r_{13}r_{34}\\ &-2r_{23}/r_{24}r_{34}-4] \qquad \ldots\ldots\ldots\ldots\ldots (14)\end{aligned}$$

The covariance between the tetrad ratios t_{12-34} and t_{13-43} is given by

$$\begin{aligned}P_{ab} = [ab/N][&1/r^2_{12}+1/r^2_{34}+2/t_{12-34}+2/t_{12-43}\\ &+r_{34}/r_{13}r_{14}+r_{12}/r_{14}r_{24}+r_{12}/r_{13}r_{23}\\ &+r_{34}/r_{23}r_{24}-r_{23}/r_{12}r_{13}-r_{14}/r_{12}r_{24}\\ &-r_{23}/r_{24}r_{34}-r_{14}/r_{13}r_{34}-r_{24}/r_{12}r_{14}-r_{13}/r_{14}r_{34}\\ &-r_{13}/r_{12}r_{23}-r_{24}/r_{23}r_{34}-2] \qquad \ldots\ldots\ldots\ldots\ldots (15)\end{aligned}$$

The covariance between the tetrad ratios t_{12-34} and t_{13-42} is given by

$$\begin{aligned}P_{ac} = [ac/N][&r_{23}/r_{12}r_{13}+r_{14}/r_{13}r_{34}+r_{14}/r_{12}r_{24}\\ &+r_{23}/r_{24}r_{34}+r_{34}/r_{13}r_{14}+r_{12}/r_{14}r_{24}\\ &+r_{12}/r_{13}r_{23}+r_{34}/r_{23}r_{24}-2t_{12-34}\\ &-2/t_{13-42}-1/r^2_{13}-1/r^2_{24}-r_{24}/r_{12}r_{14}\\ &-r_{13}/r_{14}r_{34}-r_{13}/r_{12}r_{23}-r_{24}/r_{23}r_{34}\\ &-2] \qquad \ldots\ldots\ldots\ldots\ldots (16)\end{aligned}$$

The other covariance necessary to evaluate (13) is the covariance between t_{12-43}, and t_{13-42}. This is given by

$$\begin{aligned}P_{bc} = [bc/N][&r_{23}/r_{12}r_{13}+r_{14}/r_{13}r_{34}+r_{14}/r_{12}r_{24}\\ &+r_{23}/r_{24}r_{34}+1/r^2_{14}+1/r^2_{23}+2t_{12-43}\\ &+2/t_{13-42}-r_{34}/r_{13}r_{14}-r_{12}/r_{13}r_{23}\\ &-r_{12}/r_{14}r_{24}-r_{34}/r_{23}r_{24}-r_{24}/r_{12}r_{14}\\ &-r_{13}/r_{14}r_{34}-r_{13}/r_{12}r_{23}-r_{24}/r_{23}r_{34}\\ &-2] \qquad \ldots\ldots\ldots\ldots\ldots (17)\end{aligned}$$

Substituting these values in equation (13) gives the desired variance of the function $\beta_1\beta_2/\alpha_1\alpha_2$. The derivations of formulas (13), (14), (15), (16), and (17) are given in Appendix IV. In order to evaluate the amount contributed to the covariance by each of the group factors β_1 and β_2, it is necessary to have a third independent measure of b. The analysis for such a system is given in Appendix V.*

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* Dr. Irving Lorge of Teachers College independently arrived at a similar analysis.

It has been demonstrated that the data will fit a factor pattern such as given on pages 35-38.*

Since it is the simplest pattern that fits the data it shall be assumed that the tenable pattern is the true pattern arguing from the principle of parsimony.

On this assumption then it is possible to analyze the general and group factors so as to ascribe to each of them specific amounts of the covariance.

In Table XXXVII is given the number-verbal ratio for the various stocks and the probable errors of certain of these ratios. These ratios are expressed as decimals in the table but can be thought of as percentages if desired. $\alpha_1\alpha_2$ represents the general factor of the pattern, here verbal, and $\beta_1\beta_2$ the group factor number, hypothesized in Arithmetic Computation and Arithmetic Reasoning.

The ratios in column one, Table XXXVII, were determined by formula (12) as this is more adaptable for determining the standard error. The errors were calculated from formula (13). Significant differences in the ratios are only possible in the Chinese, Portuguese, Part-Hawaiians, and Hawaiians.

As a check on the numerical value of $\beta_1\beta_2/\alpha_1\alpha_2$, these ratios were determined by a second method. These results are given in column 4. When determined from the tetrad ratios all six intercorrelations from four tests are involved. The check involves only four different correlation coefficients. From formula (9) obtain $\beta_1\beta_2$, and by subtraction from r_{12} get $\alpha_1\alpha_2$. The values for $\alpha_1\alpha_2$ are recorded in column 2, and for $\beta_1\beta_2$ in column 3. When the ratios obtained by the two techniques are compared the meth difference is .01. This difference is satisfactory in view of ane errors of measurement involved in the correlations.

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* The Chinese, however, may be represented by the pattern,

$$\begin{aligned} x_1 &= c_1a + k_1b + l_1d + e_1 \\ x_2 &= c_2a + k_2b + e_2 \\ x_3 &= c_3a + e_3 \\ x_4 &= c_4a + l_4d + e_4 \\ x_5 &= c_5a + e_5 \end{aligned}$$

If the criterion of significance of difference had been 3σ then such a pattern as hypothesized for all other stocks would have been found tenable for the Chinese.

TABLE XXXVII

THE NUMBER VERBAL RATIO FOR THE RACIAL GROUPS DETERMINED BY TWO METHODS. THE ORIGINAL VALUES FOR $\alpha_1\alpha_2$, $\beta_1\beta_2$ WERE CALCULATED TO FOUR PLACES

Race	$\frac{\beta_1\beta_2}{\alpha_1\alpha_2}$		$\alpha_1\alpha_2$*	$\beta_1\beta_2$*	$\frac{\beta_1\beta_2^*}{\alpha_1\alpha_2}$
Japanese, M.	.60		.40	.25	.61
Japanese, F.	.53		.42	.23	.54
Chinese, M.	.28	±.06	.56	.15	.27
Chinese, F.	.61	±.08	.44	.27	.62
Portuguese, M.	.76	±.16	.38	.29	.76
Portuguese, F.	.28	±.09	.56	.19	.34
Hawaiian, M.	.16	±.15	.51	.10	.19
Hawaiian, F.	.08	±.17	.61	.05	.08
Pt. Hawaiian, M.	.47	±.11	.43	.20	.46
Pt. Hawaiian, F.	.49	±.10	.40	.20	.51
Korean, M.	.61		.50	.28	.55
Korean, F.	.45		.44	.21	.48
Filipino, M.	—		—	—	—
Filipino, F.	.14		.67	.10	.15

* These values were determined by a second method, and the differences between the two sets of ratios lie well within the errors of measurement in the correlations.

The number verbal-ratio for the Portuguese males is 76/100. They exceed the ratio of the Chinese by 48 ± 17 units, the ratio of the Hawaiians by 60 ± 22 units, and the Part-Hawaiians by 29 ± 20 units. The first two agree with the findings in Table XXXVI, but the last, while agreeing in direction does not agree as to the significance of the difference.

This implies that although the strength of the number factor is greater for the Portuguese than for the Part-Hawaiians, the ratios of number to verbal is not significantly different for the two stocks.

The number-verbal ratio for the Chinese females exceeds that of the Portuguese females by 33 ± 12 units, the Hawaiian by 53 ± 19 units, and the Part-Hawaiians by 12 ± 13 units. In the first two cases it verifies to a significant degree the results of Table XXXVI. In the case of the Part-Hawaiians the direction of the difference is verified, but the difference itself is not significant.

APPENDIX I

BASIC TABLES OF MEANS, STANDARD DEVIATIONS, AND INTERCORRELATIONS

TABLE XX

INTERCORRELATIONS BETWEEN

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning
6. Age
7. Square of Age

For Japanese males, (N equals 876) and females (N equals 843)*

	Ar.C.	*Ar.R.*	*Par.M.*	*Sen.M.*	*Wor.M.*	*Age*	*Sq.A.*	*M*	*σ*
Ar.C.		.69934	.56807	.53000	.57000	.43082	.41180	96.41	23.89
Ar.R.	.70357		.66345	.59502	.63640	.36529	.36938	45.07	17.83
Par.M.	.56987	.67510		.70239	.76780	.26554	.26951	49.08	16.14
Sen.M.	.51214	.60156	.72385		.71769	.32182	.32550	27.03	13.49
Wor.M.	.57029	.66251	.83212	.74709		.33188	.33797	34.67	11.24
Age	.39342	.37682	.29709	.32495	.34453		.99836	143.81	12.99
Sq.A.	.39382	.37570	.29774	.32494	.34438	.99826			
M	96.82	51.68	50.90	28.36	35.54	143.89			
σ	25.05	19.69	16.98	13.65	12.01	12.91			

* Coefficients above and to the right of the diagonal are for the females, and those below and to the left of the diagonal are for the males. The means and standard deviations are given at the bottom for the males and at the right for the females.

TABLE XXI

INTERCORRELATIONS BETWEEN

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning
6. Age
7. Square of Age

For Chinese males (N equals 442), and females (N equals 453)*

	Ar.C.	*Ar.R.*	*Par.M.*	*Sen.M.*	*Wor.M*	*Age*	*Sq.A.*	*M*	*σ*
Ar.C.		.75539	.58797	.61457	.60820	.43393	.43641	98.46	24.06
Ar.R.	.73601		.60177	.60478	.62028	.38041	.38404	50.92	19.75
Par.M.	.64659	.70998		.74076	.79264	.21304	.21601	54.34	16.52
Sen.M.	.61279	.68856	.74758		.78419	.27495	.27694	30.79	14.15
Wor.M.	.59389	.68539	.81975	.78569		.29214	.29427	37.53	11.60
Age	.38735	.29547	.25108	.29155	.24929		.99825	142.75	13.13
Sq.A.	.38853	.30032	.25434	.29357	.25117	.99825			
M	99.97	57.57	55.05	31.28	37.39	144.51			
σ	23.95	20.93	17.13	14.26	12.42	13.38			

* Coefficients above and to the right of the diagonal are for the females, and those below and to the left of the diagonal are for the males. The means and standard deviations are given at the bottom for the males and at the right for the females.

TABLE XXII

INTERCORRELATIONS BETWEEN

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning
6. Age
7. Square of Age

For Portuguese males (N equals 139), and females (N equals 280)*

	Ar.C.	*Ar.R.*	*Par.M.*	*Sen.M.*	*Wor.M.*	*Age*	*Sq.A.*	*M*	*σ*
Ar.C.		.75931	.65161	.68037	.65264	.33068	.32968	78.80	24.78
Ar.R.	.70436		.65342	.63141	.62010	.26074	.26241	40.93	17.39
Par.M.	.50512	.58736		.74186	.77997	.14250	.15029	51.17	16.14
Sen.M.	.46230	.54118	.72585		.73620	.17873	.18725	28.06	13.46
Wor.M.	.43865	.53042	.80418	.73675		.20669	.21064	33.43	10.46
Age	.35316	.22346	.11061	.11522	.05279		.99845	146.73	12.83
Sq.A.	.36145	.23504	.11205	.11257	.05417	.99813			
M	80.89	46.37	47.61	27.02	31.83	148.66			
σ	25.95	17.49	16.56	12.16	12.33	11.99			

* Coefficients above and to the right of the diagonal are for the females, and those below and to the left of the diagonal are for the males. The means and standard deviations are given at the bottom for the males and at the right for the females.

TABLE XXIII

INTERCORRELATIONS BETWEEN

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning
6. Age
7. Square of Age

For Hawaiian males (N equals 81), and females (N equals 63)*

	Ar.C.	*Ar.R.*	*Par.M.*	*Sen.M.*	*Wor.M.*	*Age*	*Sq.A.*	*M*	*σ*
Ar.C.		.67344	.61626	.60658	.59478	.35622	.35819	82.51	24.06
Ar.R.	.66264		.63153	.66975	.60799	.28199	.27157	33.49	13.81
Par.M.	.59666	.61884		.64048	.71796	.19358	.19154	42.12	16.37
Sen.M.	.56388	.68394	.70356		.71854	.21375	.20582	22.13	11.95
Wor.M.	.63075	.70921	.79976	.75373		.15098	.15067	29.44	11.01
Age	.41860	.33540	.33776	.18215	.34670		.99801	147.55	12.27
Sq.A.	.41953	.33650	.33692	.18033	.34749	.99847			
M	80.42	39.04	42.72	20.51	29.45	147.19			
σ	25.78	19.25	16.77	12.53	11.39	12.59			

* Coefficients above and to the right of the diagonal are for the females, and those below and to the left of the diagonal are for the males. The means and standard deviations are given at the bottom for the males and at the right for the females.

TABLE XXIV

INTERCORRELATIONS BETWEEN

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning
6. Age
7. Square of Age

For Part-Hawaiian males (N equals 219), and females (N equals 244)*

	Ar.C.	*Ar.R.*	*Par.M.*	*Sen.M.*	*Wor.M.*	*Age*	*Sq.A.*	*M*	*σ*
Ar.C.		.64223	.55229	.48481	.57387	.39440	.39672	88.46	21.87
Ar.R.	.65632		.63845	.51892	.62707	.29030	.29062	42.98	16.04
Par.M.	.62681	.62471		.58507	.75147	.17193	.16937	53.65	15.20
Sen.M.	.60876	.56801	.76262		.70328	.16002	.15818	30.58	12.69
Wor.M.	.61857	.57695	.80687	.75855		.24306	.24218	36.76	10.23
Age	.28805	.27097	.23112	.28158	.28021		.99812	145.72	13.11
Sq.A.	.29370	.27136	.22787	.28504	.27838	.99810			
M	86.46	47.88	50.75	28.02	34.71	144.11			
σ	22.56	17.89	15.25	13.14	11.52	13.32			

* Coefficients above and to the right of the diagonal are for the females, and those below and to the left of the diagonal are for the males. The means and standard deviations are given at the bottom for the males and at the right for the females.

TABLE XXV

Intercorrelations between

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning
6. Age
7. Square of Age

For Korean males (N equals 67), and females (N equals 52)*

	Ar.C	*Ar.R.*	*Par.M.*	*Sen.M.*	*Wor.M.*	*Age*	*Sq.A.*	*M*	*σ*
Ar.C.		.70798	.63064	.64833	.49844	.45339	.45784	98.77	26.36
Ar.R.	.79923		.69831	.63979	.70070	.43495	.42983	51.08	24.36
Par.M.	.63922	.71168		.76249	.77556	.23564	.23766	58.58	15.06
Sen.M.	.64981	.60955	.69271		.74257	.46109	.45937	31.96	13.18
Wor.M.	.67253	.67743	.82936	.76070		.39531	.39215	40.12	11.23
Age	.35953	.36261	.38628	.42356	.39374		.99848	143.42	11.55
Sq.A.	.35923	.36113	.38247	.42803	.38536	.99817			
M	99.16	57.10	56.21	33.50	40.66	143.82			
σ	25.13	18.35	15.93	14.35	11.94	13.06			

* Coefficients above and to the right of the diagonal are for the females, and those below and to the left of the diagonal are for the males. The means and standard deviations are given at the bottom for the males and at the right for the females.

TABLE XXVI

Intercorrelations between

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning
6. Age
7. Square of Age

For Filipino males (N equals 25), and females (N equals 31)*

	Ar.C.	*Ar.R.*	*Par.M.*	*Sen.M.*	*Wor.M.*	*Age*	*Sq.A.*	*M*	*σ*
Ar.C.		.76547	.68746	.63253	.71641	.28694	.29300	99.03	26.00
Ar.R.	.46197		.62965	.72957	.69477	.30854	.30115	44.84	21.84
Par.M.	.67303	.66964		.65703	.87212	.38904	.37953	58.05	18.47
Sen.M.	.51780	.57299	.53811		.75120	.35985	.34813	30.53	13.74
Wor.M.	.50697	.53852	.61489	.67371		.31715	.30951	38.53	14.30
Age	.62803	.29541	.51961	.39914	.38788		.99773	146.44	11.28
Sq.A.	.64059	.31322	.52508	.41107	.40947	.99859			
M	89.68	46.16	51.70	28.86	33.66	148.10			
σ	21.64	17.54	12.81	10.83	8.87	13.41			

* Coefficients above and to the right of the diagonal are for the females, and those below and to the left of the diagonal are for the males. The means and standard deviations are given at the bottom for the males and at the right for the females.

APPENDIX II

TABLES OF PARTIAL CORRELATIONS USED IN COMPUTING THE TETRADS

TABLE XXVII

PARTIAL CORRELATION COEFFICIENTS RESULTING WHEN AGE AND SQUARE OF AGE HAVE BEEN HELD CONSTANT FOR THE JAPANESE MALES AND FEMALES*

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning

	1	*2*	*3*	*4*	*5*
1		.64599	.52113	45520	.49640
2	.65203		.63356	.54007	.58392
3	.51466	.63636		.67003	.73649
4	.44224	.54426	.69206		.67931
5	.50333	.61258	.81090	.71845	

* The coefficients below and to the left of the diagonal are for males, those above and to the right of the diagonal are for females.

TABLE XXVIII

PARTIAL CORRELATION COEFFICIENTS RESULTING WHEN AGE AND SQUARE OF AGE HAVE BEEN HELD CONSTANT FOR THE CHINESE MALES AND FEMALES*

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning

	1	*2*	*3*	*4*	*5*
1		.70934	.56352	.56954	.48173
2	.71010		.57962	.56634	.57143
3	.61218	.68323		.72783	.77914
4	.56311	.66352	.72755		.76635
5	.55769	.66355	.80982	.76780	

* The coefficients below and to the left of the diagonal are for males, those above and to the right of the diagonal are for females.

TABLE XXIX

Partial Correlation Coefficients Resulting When Age and Square of age Have Been Held Constant for the Portuguese Males and Females

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning

	1	*2*	*3*	*4*	*5*
1		.74377	.65505	.67719	.66667
2	.67273		.64726	.61724	.60133
3	.50729	.58757		.72635	.77365
4	.46939	.54648	.72629		.72449
5	.45058	.53933	.80811	.74054	

* The coefficients below and to the left of the diagonal are for males, those above and to the right of the diagonal are for females.

TABLE XXX

Partial Correlation Coefficients Resulting When Age and Square of Age Have Been Held Constant for the Hawaiian Males and Females*

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning

	1	*2*	*3*	*4*	*5*
1		.65909	.59452	.59167	.58583
2	.60687		.61351	.64384	.60590
3	.52672	.57353		.62698	.68394
4	.54945	.67254	.69366		.71916
5	.50000	.67037	.77407	.75177	

* The coefficients below and to the left of the diagonal are for males, those above and to the right of the diagonal are for females.

TABLE XXXI

PARTIAL CORRELATION COEFFICIENTS RESULTING WHEN AGE AND SQUARE OF AGE HAVE BEEN HELD CONSTANT FOR THE PART-HAWAIIAN MALES AND FEMALES*

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning

	1	*2*	*3*	*4*	*5*
1		.59819	.53666	.46628	.54030
2	.62931		.62817	.50141	.60172
3	.60795	.60112		.67945	.74373
4	.56936	.53143	.75141		.69638
5	.59538	.54571	.79944	.74425	

* The coefficients below and to the left of the diagonal are for males, those above and to the right of the diagonal are for females.

TABLE XXXII

PARTIAL CORRELATION COEFFICIENTS RESULTING WHEN AGE AND SQUARE OF AGE HAVE BEEN HELD CONSTANT FOR THE KOREAN MALES AND FEMALES*

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5 Word Meaning

	1	*2*	*3*	*4*	*5*
1		.65560	.61154	.56067	.40244
2	.77287		.69318	.54959	.64000
3	.58599	.66561		.76336	.77407
4	.39935	.54545	.64262		.69231
5	.62903	.62903	.80130	.70064	

* The coefficients below and to the left of the diagonal are for males, those above and to the right of the diagonal are for females.

TABLE XXXIII

PARTIAL CORRELATION COEFFICIENTS RESULTING WHEN AGE AND SQUARE OF AGE HAVE BEEN HELD CONSTANT FOR THE FILIPINO MALES AND FEMALES*

1. Arithmetic Computation
2. Arithmetic Reasoning
3. Paragraph Meaning
4. Sentence Meaning
5. Word Meaning

	1	*2*	*3*	*4*	*5*
1		.76658	.56997	.63010	.70762
2	.28108		.57692	.70103	.66005
3	.51412	.62150		.59893	.85861
4	.32065	.47297	.40566		.74742
5	.26316	.38647	.51515	.58252	

* The coefficients below and to the left of the diagonal are for males, those above and to the right of the diagonal are for females.

APPENDIX III

THE STANDARD ERROR OF THE MEAN OF TETRADS OF THE TYPE t_{12ij}

The mean of tetrads of the type t_{12ij} is used in several types of analysis. Tetrads t_{12ij} all have variables x_1 and x_2 as the identifying subscripts, and x_i and x_j take the subscripts of the other variables.

The mean involves $(n-2)(n-3)$ tetrads where n is the number of variables. Now since every tetrad is correlated with every other tetrad, the ordinary formula for the standard error of the mean of $(n-2)(n-3)$ correlated measures is an expression of the order $(n-2)^2(n-3)^2$ terms, each term being a polynomial of four terms. All of the expanded terms, except the square terms, involve correlations between correlations which of themselves are lengthy and laborious to calculate. In the five variable problem which was considered there were seventy-seven different terms requiring such a lengthy calculation.

Clearly it is out of the question to calculate such a standard error without approximations. If the correlations lie substantially between .00 and .80 and are fairly uniform for a given table one may expect fairly close results from classifying terms by type and resorting to the following approximations. The following analysis follows Kelley (1928) very closely in assumptions necessary to secure reasonable approximations but differs in its further analysis of certain terms.

n, the number of variables

r_{12}, the correlation between x_1 and x_2

r_1, the mean correlation between x_1 and any variable not x_2, and of that between x_2 and any variable not x_1
$r_1 = \Sigma(r_{1i}+r_{2i})/2(n-2)$

r, the mean correlation between two variables neither of which is x_1 or x_2.
$r = 2\Sigma(r_{ij})/(n-2)(n-3)$

σ^2, the mean of the variances of the correlation coefficients.

q, the mean product-moment of the errors in two correlation coefficients which involve three variables only.

p, the mean product-moment of the errors in two correlation coefficients which involve four variables.

N, the population.

Let t_{12} = mean tetrad of t_{12ij}, and t_{12ji} tetrads.

$$t_{12} = \frac{1}{(n-2)(n-3)} \left\{ \begin{array}{l} \quad t_{12ij} + t_{12ji} \\ + t_{12ik} + t_{12ki} \\ + \quad\; + \\ + t_{12kl} + t_{12lk} \\ + \quad\; + \\ + \quad\; + \end{array} \right.$$

Let Δ_{12} represent an error in t_{12}, etc. Then

$$\Delta_{12ij} = \frac{1}{(n-2)(n-3)} \left\{ \begin{array}{l} \Delta_{12ij} + \Delta_{12ji} \\ + \Delta_{12ik} + \Delta_{12ki} \\ + \quad\; + \\ + \Delta_{12kl} + \Delta_{12lk} \\ + \quad\; + \end{array} \right.$$

Squaring both sides, summing and dividing by the number of samples

$$\sigma^2_{t_{12}} = \left(\frac{1}{U}\right)\left(\frac{1}{(n-2)^2(n-3)^2}\right. \left\{ \begin{array}{l} \quad (n-2)(n-3)\Sigma\Delta^2_{12ij} \\ + (n-2)(n-3)\Sigma\Delta_{12ij}\Delta_{12ji} \\ + 2(n-2)(n-3)(n-4)\Sigma\Delta_{12ij}\Delta_{12ik} \\ + 2(n-2)(n-3)(n-4)\Sigma\Delta_{12ij}\Delta_{12ki} \\ + (n-2)(n-3)(n-4)(n-5)\Sigma\Delta_{12ij}\Delta_{12kl} \end{array} \right.$$

where U is the universe.

Let us designate the summation terms in the bracket successively A, B, C, D, and E, when they have been divided by U

$$\sigma^2_{t_{12}} = [1/(n-2)(n-3)][A + B + 2(n-4)C + 2(n-4)D + (n-4)(n-5)E]$$

$$A = \tfrac{1}{U}\Sigma\Delta^2_{12ij} = \sigma^2_{12ij} = \tfrac{1}{U}\Sigma(r_{12}\delta_{ij} + r_{ij}\delta_{12} - r_{1i}\delta_{2j} - r_{2j}\delta_{1i})^2$$

$$\begin{aligned} A = \tfrac{1}{U}(&r^2_{12}\Sigma\delta^2_{ij} + r^2_{ij}\Sigma\delta^2_{12} + r^2_{1i}\Sigma\delta^2_{2j} + r^2_{2j}\Sigma\delta^2_{1i} + 2r_{12}r_{ij}\Sigma\delta_{12}\delta_{1j} \\ &- 2r_{12}r_{1i}\Sigma\delta_{2j}\delta_{ij} - 2r_{12}r_{2j}\Sigma\delta_{1i}\delta_{ij} - 2r_{1i}r_{ij}\Sigma\delta_{12}\delta_{2j} \\ &- 2r_{2j}r_{ij}\Sigma\delta_{12}\delta_{1i} + 2r_{1i}r_{2j}\Sigma\delta_{1i}\delta_{2j}) \end{aligned}$$

and define $p_{r_{ij}r_{k1}}$ as p_{ijk1}

$$\begin{aligned}A=\sigma^2_{12ij}=r^2_{12}\sigma^2_{ij}+r^2_{ij}\sigma^2_{12}+r^2_{1i}\sigma^2_{2j}+r^2_{2j}\sigma^2_{1i}+2r_{12}r_{ij}p_{12ij}\\+2r_{1i}r_{2j}p_{1i2j}-2r_{12}r_{1i}p_{2jij}-2r_{12}r_{2j}p_{1i\,ij}\\-2r_{1i}r_{ij}p_{122j}-2r_{2j}r_{ij}p_{121i}\end{aligned}$$

and find similarly

$$\begin{aligned}B=r^2_{12}\sigma^2_{ij}+r^2_{ij}\sigma^2_{12}+r_{1i}r_{1j}p_{2i2j}+r_{2i}r_{2j}p_{1i1j}\\+2r_{12}r_{ij}p_{12ij}+r_{1i}r_{2i}p_{1j2j}+r_{1j}r_{2j}p_{1i2i}\\-r_{12}r_{1j}p_{2iij}-r_{12}r_{2i}p_{1jij}-r_{1j}r_{ij}p_{122i}-r_{2i}r_{ij}p_{121j}\\-r_{12}r_{1i}p_{2jij}-r_{12}r_{2j}p_{1iij}-r_{1i}r_{ij}p_{122j}-r_{2j}r_{ij}p_{121i}\end{aligned}$$

and similarly

$$\begin{aligned}C=r^2_{12}p_{ijik}+r_{ij}r_{ik}\sigma^2_{12}+r^2_{1i}p_{2j2k}+r_{2j}r_{2k}\sigma^2_{1i}+r_{12}r_{ij}p_{12ik}\\+r_{12}r_{ik}p_{12ij}+r_{1i}r_{2k}p_{1i2j}+r_{1i}r_{2j}p_{1i2k}-r_{12}r_{1i}p_{2kij}\\-r_{12}r_{2k}p_{1iij}-r_{1i}r_{ij}p_{122k}-r_{2k}r_{ij}p_{121i}-r_{12}r_{1i}p_{2jik}\\-r_{12}r_{2j}p_{1iik}-r_{1i}r_{ik}p_{122j}-r_{2j}r_{ik}p_{121i}\end{aligned}$$

and similarly

$$\begin{aligned}D=r^2_{12}p_{ijik}+r_{ij}r_{ik}\sigma^2_{12}+r_{1i}r_{1k}p_{2i2j}+r_{2i}r_{2j}p_{1i1k}\\+r_{12}r_{ik}p_{12ij}+r_{12}r_{ij}p_{12ik}+r_{1i}r_{2i}p_{1k2j}+r_{1k}r_{2j}p_{1i1k}\\-r_{12}r_{ik}p_{2iij}-r_{12}r_{2i}p_{1kij}-r_{1k}r_{ij}p_{122i}-r_{2i}r_{ij}p_{121k}\\-r_{12}r_{1i}p_{2jik}-r_{12}r_{2j}p_{1iik}-r_{1i}r_{ik}p_{122j}-r_{2j}r_{ik}p_{121i}\end{aligned}$$

and similarly

$$\begin{aligned}E=r^2_{12}p_{ijkl}+r_{ij}r_{kl}\sigma^2_{12}+r_{1i}r_{1k}p_{2j2l}+r_{2j}r_{2l}p_{1i1k}\\+r_{12}r_{kl}p_{12ij}+r_{12}r_{ij}p_{12kl}+r_{1i}r_{2l}p_{1k2j}+r_{1k}r_{2j}p_{1i2l}\\-r_{12}r_{1k}p_{2lij}-r_{12}r_{2l}p_{1kij}-r_{1k}r_{ij}p_{122l}-r_{2l}r_{ij}p_{121k}\\-r_{12}r_{1i}p_{2jkl}-r_{12}r_{2j}p_{1ikl}-r_{1i}r_{kl}p_{122j}-r_{2j}r_{kl}p_{121i}\end{aligned}$$

Now distribute these values in vertical arrays according to their respective headings, remembering the appropriate coefficients.

TABLE I

SUMMARY OF THE COEFFICIENTS OF EXPRESSIONS A, B, C, D AND E

Terms	*A*	*B*	*C*	*D*	*E*	Σ
$r_{12}^2\sigma^2$	1	1				2
$r_1^2\sigma^2$	2		1			$2(n-3)$
$r^2\sigma^2$	1	1	1	1	1	$(n-2)(n-3)$
r_{12}^2q			1	1		$4(n-4)$
r_{12}^2p					1	$(n-4)(n-5)$
$-r_{12}r_1q$	4	4	2	2		$8(n-3)$
$-r_{12}r_1p$			2	2	4	$4(n-3)(n-4)$
$r_{12}rp$	2	2	2	2	2	$2(n-2)(n-3)$
r_1^2q		4	1	3	2	$2(n-2)(n-3)$
r_1^2p	2		2	1	2	$2(n-3)^2$
$-r_1rq$	4	4	4	4	4	$4(n-2)(n-3)$

Summing each of these five expressions, taking each the appropriate number of times, i.e., to one A there is one B, $2(n-4)$ C's, $2(n-4)$ D's and $(n-4)(n-5)$ E's, the variance of t_{12} is obtained.

$$\begin{aligned}\sigma^2_{t_{12}} = {}&[1/(n-2)(n-3)][2r_{12}^2\sigma^2+2(n-3)r_1^2\sigma^2+(n-2)(n-3)r^2\sigma^2\\ &+4(n-4)r_{12}^2q+(n-4)(n-5)r_{12}^2p-8(n-3)r_{12}r_1q\\ &-4(n-3)(n-4)r_{12}r_1p+2(n-2)(n-3)r_{12}rp\\ &+2(n-2)(n-3)r_1^2q+2(n-3)^2r_1^2p-4(n-2)(n-3)r_1rq] \qquad \ldots\ldots\ldots\ldots(8)\end{aligned}$$

Now in Table II write this equation in vertical arrays, supplying numerical values for the coefficients given by the terms for different values of n.

It is now necessary to define specifically how the various terms used in this formula shall be calculated. r_{12}, r_1, and r have been defined previously. The variance of a correlation coefficient is σ^2. Thus, a single value is needed to substitute for a large number of slightly different values. In calculating the standard error of the mean tetrad t_{12}, r_{12} enters as often as r_{ij}, and the two together as often as r_{1i} and r_{2i}. Therefore, define

$$R=[1/4](r_{12}+2r_1+r)$$

and

$$\sigma^2=(1-R^2)^2/N$$

TABLE II

THE VALUES OF THE COEFFICIENTS OF THE TERMS IN FORMULA 8 FOR PARTICULAR n'S

Term *n*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
$r_{12}^2\sigma^2$	2	2	2	2	2	2	2	2	2	2	2
$r_1^2\sigma^2$	4	6	8	10	12	14	16	18	20	22	24
$r^2\sigma^2$	6	12	20	30	42	56	72	90	110	132	156
r_{12}^2q	4	8	12	16	20	24	28	32	36	40	44
r_{12}^2p	0	2	6	12	20	30	42	56	72	90	110
$-r_{12}r_1q$	16	24	32	40	48	56	64	72	80	88	96
$-r_{12}r_1p$	8	24	48	80	120	168	224	288	360	440	528
$r_{12}rp$	12	24	40	60	84	112	144	180	220	264	312
r_1^2q	12	24	40	60	84	112	144	180	220	264	312
r_1^2p	8	18	32	50	72	98	128	162	200	242	288
$-r_1rq$	24	48	80	120	168	224	288	360	440	528	624
$1/(n-2)(n-3)$	1/6	1/12	1/20	1/30	1/42	1/56	1/72	1/90	1/110	1/132	1/156

The product moment q which involves three variables (p_{121i}) may be calculated by the formula

$$p_{1213} = r_{23} - 1/2(1 - r_{12}^2 - r_{13}^2 - r_{23}^2 + 2r_{12}r_{13}r_{23})$$

Obviously there are a great many values of q, depending on the three correlation coefficients. Consider the tetrad t_{12ij}, from which sets of three variables 12i, 12j, 1ij, and 2ij are obtained. Variables 1 and 2 enter equally with i and j, and it now remains to choose x_i from the remaining $n-2$ variables. Now following Kelley's (1928) reasoning:

"Thus, if $(r_{1i}+r_{2i})/2$ is calculated for each of the $(n-2)$ variables, and if the variables are arranged in order upon the basis of these magnitudes, the median of them will presumably be most nearly equal to r_1. This will determine the particular variable of the $(n-2)$ which is called x_i for the purpose of calculating q. After determining the variables we still have three values of q, depending upon the order in which they are taken . . . If q takes quite different values, depending upon which of the three orders (a) x_i, x_1, x_2, (b) x_1, x_i, x_2, (c) x_2, x_i, x_1 is taken, use for q the average of the three. If inspection informs one which of these three values of q will be the median, then that one may be taken in place of the mean of the three, thus shortening the calculation." P. 91.

In this particular problem where n is small, all possible values of q were determined so that to that extent no approximation occurs.

The product moment designated p involves four variables (p_{ijkl}). Now two of these variables will be x_1 and x_2. The other two x_i and x_j, should represent a fair sampling of the $(n-2)$ variables.

Now choose them on the following basis. Arrange the $n-2$ variables in order of the magnitudes of the quantities $(r_{1i}+r_{2i})/2$. Then choose the variable which most nearly yields the upper and lower quartile values as x_i and x_j. If p takes quite different values, depending on the order used, i.e. (a) x_1, x_2, x_i, x_j, (b) x_1, x_i, x_2, x_j, or (c) x_1, x_j, x_2, x_i use the average of the three for p. When, as in this problem only five variables are involved, it is better to calculate all the possible p's and take the average of these.

The product moment p_{1234} is given by the following formula

$$p_{1234} = [1/N][r_{13}r_{24} + r_{14}r_{23} - (r_{12}r_{13}r_{14} + r_{12}r_{23}r_{24} + r_{13}r_{23}r_{34} + r_{14}r_{24}r_{34}) + (r_{12}r_{34}/2)(r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2)]$$

It is advisable to work out a job analysis sheet if there is any considerable number of product moments to be calculated. Kelley (op. cit.) offers an ingenious method for labor saving on the p_{121i} types. Several set-ups can be made for the p_{12ij} types, the best one depending on the type of calculating machines available. A job analysis sheet for $\sigma^2_{t_{12}}$ is extremely useful, as all functions of a type may be calculated at one time, thus decreasing errors, as well as facilitating checking.

APPENDIX IV

THE VARIANCE OF A TETRAD RATIO, AND THE COVARIANCES BETWEEN TETRAD RATIOS FROM THE SAME FOUR VARIABLES

In determining the variance and the covariance between two tetrad ratios the use of product moments facilitates derivation. Wishart (1928) has given the variances of certain product moments. Formulas (1), (2), and (3) are taken from his work with the following modifications: Where Wishart gives $(N-1)/N^2$, substitute the value N; further, replace his population values with the sample values.

$$N\sigma^2_{p_{12}} = \sigma_1^2\sigma_2^2 + p_{12}^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$Np_{p_{12}p_{13}} = \sigma_1^2 p_{23} + p_{12}p_{13} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$Np_{p_{12}p_{34}} = p_{13}p_{24} + p_{14}p_{23} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Now take successively the logarithmic differentials of t_{12-34}, t_{12-43}, and t_{13-42} after first writing them in terms of product moments. This last has no effect on either the value or the sampling distribution.

$$dt_{12-34}/t_{12-34} = dp_{12}/p_{12} + dp_{34}/p_{34} - dp_{13}/p_{13} - dp_{24}/p_{24} \ldots\ldots (4)$$

$$dt_{12-43}/t_{12-43} = dp_{12}/p_{12} + dp_{34}/p_{34} - dp_{14}/p_{14} - dp_{23}/p_{23} \ldots\ldots (5)$$

$$dt_{13-42}/t_{13-42} = dp_{13}/p_{13} + dp_{24}/p_{24} - dp_{14}/p_{14} - dp_{23}/p_{23} \ldots\ldots (6)$$

Multiplying equations (4) and (5), summing, and dividing by the number of samples gives, after applying equations (1), (2), and (3):

$$\begin{aligned} p_{ab} = [ab/N](&1/r_{12}^2 + 1/r_{34}^2 + 2/t_{12-34} + 2/t_{12-43} \\ &+ r_{34}/r_{13}r_{14} + r_{12}/r_{14}r_{24} + r_{12}/r_{13}r_{23} \\ &+ r_{34}/r_{23}r_{24} - r_{23}/r_{12}r_{13} - r_{14}/r_{12}r_{24} \\ &- r_{23}/r_{24}r_{34} - r_{14}/r_{13}r_{34} - r_{24}/r_{12}r_{14} - r_{13}/r_{14}r_{34} \\ &- r_{13}/r_{12}r_{23} - r_{24}/r_{23}r_{34} - 2) \ldots\ldots\ldots\ldots\ldots\ldots (7) \end{aligned}$$

(The covariance between the tetrad ratios t_{12-34} and t_{12-43}.) Multiplying equations (4) and (6), summing, and dividing by the number of samples gives, after applying equations (1), (2), and (3):

$$\begin{aligned}
p_{ac} = [ac/N](&r_{23}/r_{12}r_{13} + r_{14}/r_{13}r_{34} + r_{14}/r_{12}r_{24} + \\
&r_{23}/r_{24}r_{34} + r_{34}/r_{13}r_{14} + r_{12}/r_{14}r_{24} + \\
&r_{12}/r_{13}r_{23} + r_{34}/r_{23}r_{24} - 2t_{12-34} - \\
&2/t_{13-42} - 1/r_{13}^2 - 1/r_{24}^2 - r_{24}/r_{12}r_{14} - \\
&r_{13}/r_{14}r_{34} - r_{13}/r_{12}r_{23} - r_{24}/r_{23}r_{34} - \\
&2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8)
\end{aligned}$$

(The covariance between the tetrad ratios t_{12-34} and t_{13-42}.) Similarly, multiplying equations (5) and (6), summing, and dividing by the number of samples gives, after applying equations (1), (2), and (3):

$$\begin{aligned}
p_{bc} = [bc/N](&r_{23}/r_{12}r_{13} + r_{14}/r_{13}r_{34} + r_{14}/r_{12}r_{24} + \\
&r_{23}/r_{24}r_{34} + 1/r_{14}^2 + 1/r_{23}^2 + 2t_{12-43} + \\
&2/t_{13-42} - r_{34}/r_{13}r_{14} - r_{12}/r_{13}r_{23} - \\
&r_{12}/r_{14}r_{24} - r_{34}/r_{23}r_{24} - r_{24}/r_{12}r_{14} - \\
&r_{13}/r_{14}r_{34} - r_{13}/r_{12}r_{23} - r_{24}/r_{23}r_{34} - \\
&2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (9)
\end{aligned}$$

(The covariance between the tetrad ratios t_{12-43} and t_{13-42}). Substituting these values in equation (13) gives us the desired variance of the function $\beta_1\beta_2/\alpha_1\alpha_2$. The numerical solution is fairly rapid if the various terms are written in a column and then in successive columns headed σ_a^2, σ_b^2, σ_c^2, p_{ac}, and p_{bc} write the appropriate coefficients and signs. Then by summation of the various columns we have the numerical values to substitute in equation (13).

Squaring equation (4), summing, and dividing by the number of samples gives, after applying equations (1), (2), and (3):

$$\begin{aligned}
\sigma_{t_{12-34}}^2 = [t_{12-34}^2/N][&1/r_{12}^2 + 1/r_{13}^2 + 1/r_{24}^2 + 1/r_{34}^2 + \\
&2/t_{12-34} + 2t_{13-42} + 2/t_{12-43} + \\
&2/t_{13-42} - 2r_{23}/r_{12}r_{13} - 2r_{14}/r_{12}r_{24} - \\
&2r_{14}/r_{13}r_{34} - 2r_{23}/r_{24}r_{34} - 4] \ldots\ldots\ldots\ldots (10)
\end{aligned}$$

APPENDIX V

THE ANALYSIS OF A PARTICULAR PATTERN FOR FIVE VARIABLES

Proposition V: Given five variables with a general factor common to all, and a group factor contained in three variables, we can analyze the covariance so as to ascribe specific amounts to each of the factors involved.

$$\begin{aligned}
x_1 &= c_1a + k_1b + e_1 \\
x_2 &= c_2a + k_2b + e_2 \\
x_3 &= c_3a + k_3b + e_3 \\
x_4 &= c_4a \qquad\quad + e_4 \\
x_5 &= c_5a \qquad\quad + e_5
\end{aligned}$$

Let the α's and β's be defined as in Chapter IV, then

$$\begin{aligned}
r_{12} &= \alpha_1\alpha_2 + \beta_1\beta_2 & r_{24} &= \alpha_2\alpha_4 \\
r_{13} &= \alpha_1\alpha_3 + \beta_1\beta_3 & r_{25} &= \alpha_2\alpha_5 \\
r_{14} &= \alpha_1\alpha_4 & r_{34} &= \alpha_3\alpha_4 \\
r_{15} &= \alpha_1\alpha_5 & r_{35} &= \alpha_3\alpha_5 \\
r_{23} &= \alpha_2\alpha_3 + \beta_2\beta_3 & r_{45} &= \alpha_4\alpha_5
\end{aligned}$$

Now

$$\begin{aligned}
\alpha_1^2 &= r_{14}r_{15}/r_{45} \\
\alpha_2^2 &= r_{24}r_{25}/r_{45} \\
\alpha_3^2 &= r_{34}r_{35}/r_{45} \\
\alpha_4^2 &= r_{14}r_{45}/r_{15} \\
\alpha_5^2 &= r_{35}r_{45}/r_{34}
\end{aligned}$$

$$\beta_1\beta_2 = r_{12} - (r_{14}r_{15}r_{24}r_{25})^{1/2}/r_{45} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

or alternatively from t_{1245} we have

$$\beta_1\beta_2 = t_{1245}/r_{45}$$

$$\beta_1\beta_3 = r_{13} - (r_{14}r_{15}r_{34}r_{35})^{1/2}/r_{45} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

or alternatively from tetrad t_{1345}

$$\beta_1\beta_3 = t_{1345}/r_{45}$$

$$\beta_2\beta_3 = r_{23} - (r_{24}r_{25}r_{34}r_{35})^{1/2}/r_{45} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

or alternatively from tetrad t_{2345}

$$\beta_2\beta_3 = t_{2345}/r_{45}$$

We now have sufficient data to solve for β_1, β_2, and β_3. In solving for β_1, β_2, and β_3 we should choose our equations all from the first, or all from the alternative choices. Now from the alternative set of three equations we get

$$\begin{aligned}\beta_1^2 &= (t_{1245}t_{1345})/r_{45}t_{2345}\\ \beta_2^2 &= (t_{1245}t_{2345})/r_{45}t_{1345} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (30)\\ \beta_3^2 &= (t_{1345}t_{2345})/r_{45}t_{1245}\end{aligned}$$

This technique was not used in this study because a group factor ran through only two of the five tests. Thus it was not possible here to ascribe the amount of the covariance due to the group factor in each variable.

Analogous systems may be worked out with the tetrad ratio, which give the same numerical results.

APPENDIX VI

SEX DIFFERENCES AND IDIOSYNCRASIES

In Arithmetic Computation no significant sex difference is found in any group. In Arithmetic Reasoning, however, the males always exceed the females, and significantly so for the Japanese, Chinese, Portuguese, Hawaiians, and Part-Hawaiians. The Korean males just fail to meet the statistical criterion (of being 3 P. E. greater) and the Filipino males while exceeding the Filipino females are not significantly better. The probabilities of these findings, however, since they all run in the same direction can be compounded.*

That the males in general exceed the females in Arithmetic Reasoning as measured by the Stanford Achievement Test is an inference supported by the consistency of the data. In the verbal tests when the seven groups are considered there is no evidence of a general sex difference.** All other sex differences in the verbal tests are apparently negligible.

From Table XXXVII it may be seen that the number-verbal ratio for the Portuguese males exceeds that of the Portuguese females by 48 ± 19 units. The Chinese females, however, exceed the Chinese males by 33 ± 10 units in the number-verbal ratio. No other sex idiosyncrasies are apparent from the table and it would seem, therefore, that these idiosyncrasies are not to be explained as due to the sex factor, but are inherent to the stocks.

The Portuguese males excel the females in the relative strength of the number factor though in actual test scores they are not consistently better.

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* The logical criterion of consistency of direction of difference supports the inferences deduced from the statistical treatment of the data. Even when the statistical criterion is not met for a particular stock the fact that the direction of difference is consistent with that of the rest of the data will support generalized inferences.

** In but three instances out of twenty-one are differences found. In one case the males significantly exceed the females and in two cases the females significantly exceed the males. In Paragraph Meaning the Japanese males score significantly higher than the Japanese females, whereas Portuguese females score higher than the Portuguese males. In Sentence Meaning the Part-Hawaiian females score higher than the Part-Hawaiian males.

The Chinese females although they have a relatively stronger number factor than the Chinese males, make lower absolute scores on the number tests.

APPENDIX VII

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